Quantum Feedback Control of a Solid-State Qubit

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The principle of feedback control is used in a wide variety of physical and engineering problems. For example, it can be applied in a straightforward way to tune the oscillation phase of a harmonic oscillator in order to achieve a desired synchronization. An intriguing and fundamental question is whether continuous feedback can be used to control quantum systems; for instance, if it is possible or not to tune the phase of quantum (Rabi) oscillations of a two-level system (qubit).

At first sight the quantum feedback seems to be impossible because according to the "orthodox" collapse postulate [1] the quantum state is abruptly destroyed by the act of measurement. However, in a typical solid-state realization the measurement is not instantaneous but rather a continuous process (because of weak coupling and finite noise of a detector), hence, the collapse postulate is not directly applicable. The possibility of continuous feedback control of an individual qubit has been recently shown theoretically [2] using the Bayesian formalism [3,4] developed to describe the evolution of an individual quantum system. (The conventional ensemble-averaged formalism [5] cannot be used to describe quantum feedback.) The goal of the present work is a more detailed study of the continuous feedback control of quantum oscillations of a qubit state.

As a particular example we consider a qubit based on the double-quantum-dot occupied by a single electron. The qubit state (electron position in either first or second dot) is continuously measured by a weakly coupled Quantum Point Contact (QPC) nearby, so that the noisy QPC current $I(t)$ contains the information on the qubit evolution. Even though detector signal $I(t)$ does not have one-to-one correspondence with the diagonal elements of the qubit density matrix $\rho_{ij}(t)$ (this would contradict uncertainty principle), the evolution of ρ can be monitored exactly plugging $I(t)$ into Bayesian equations. Then the deviation from the desired qubit evolution can be continuously compensated by the feedback loop, which controls the energy asymmetry ε and/or the tunneling strength H of the qubit [the qubit Hamiltonian is $H_{QB}=(\varepsilon/2)(c_1^+c_1 - c_2^+c_2)+H(c_1^+c_2 + c_2^+c_1)$].

We have analyzed the operation of such quantum feedback loop using Monte-Carlo simulation of the measurement process [2]. Figure 1 shows the numerically calculated correlation function $K_I(\tau)\equiv\langle I(t)I(t+\tau)\rangle$ of the detector current for three values of the dimensionless feedback strength: F=0.0, 0.03, and 0.3. In this example the feedback signal controls the barrier height between quantum dots (we assume ε=0) using the linear relation $\Delta H/H=F\,\Delta\varphi$, where $\Delta\varphi$ is the phase difference between actual and desired quantum oscillations of a qubit state. The normalization of $K_I(\tau)$ is chosen in a way that the "perfect" signal $I(t) = I_0 + (\Delta I/2)\cos\Omega t$ would correspond to oscillations with amplitude equal to unity (here $\Delta I=I_1-I_2$ is the difference between average currents corresponding to qubit states |1> and |2>).

In absence of the feedback control, F=0, the correlation function decays in time exponentially according to the qubit dephasing rate $\Gamma= (\Delta I)^2/4S$, where S is the spectral density of the QPC shot noise. The Q-factor of oscillations is determined by the coupling $\alpha\equiv\hbar(\Delta I)^2/8SH$ between the double-dot and QPC ($Q=\alpha^{-1}$=8 in Fig. 1). As expected, the feedback synchronizes the quantum oscillations leading to nonvanishing amplitude of $K_I(\tau)$ at arbitrary long τ. This asymptotic amplitude depends on the feedback strength and becomes close to 1 in units of $(\Delta I)^2/8$ (perfect oscillations) at $F\gg\alpha$ (see Fig. 1). Analytical results obtained in this regime are in good agreement with numerical calculations. The nonvanishing oscillations of $K_I(\tau)$ lead to a δ-like peak in the spectral density of the detector current $I(t)$ at the desired frequency Ω [which is chosen coinciding with $(4H^2+\varepsilon^2)^{1/2}/\hbar$] and also change the peak-like "pedestal" in comparison with the case without feedback [6]. It is interesting to notice that the value of $K_I(+0)$ which in absence of feedback is twice larger than for perfect oscillations (see discussion of this nonclassical effect in [6]), does not change when the feedback control is applied.

We have also studied the case of moderately large coupling, $\alpha\sim 1$, and confirmed the presence of long-range order as revealed by the nonvanishing oscillations of $K_I(\tau)$. The effect of extra dephasing due to environment and the suppression of this dephasing by using quantum feedback have been also studied quantitatively by analyzing correlation functions of the detector current and the qubit density matrix.

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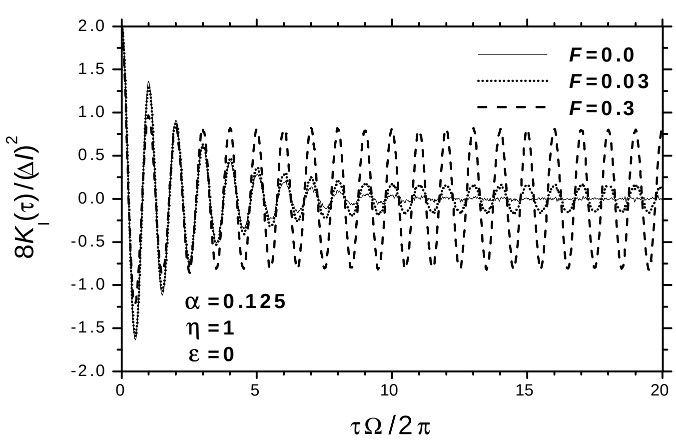

Fig. 1. Correlation function $K_I(\tau)$ of the QPC current $I(t)$ in absence of quantum feedback (solid line) and for feedback strength F=0.03 (dotted line) and 0.3 (dashed line). The feedback loop maintains quantum oscillations of the qubit state with frequency $\Omega=(4H^2+\varepsilon^2)^{1/2}/\hbar$.