SWITCHING EFFECTS OF A SINGLE THREE-LEVEL ATOM WITHIN PHOTONIC CRYSTALS FOR QUANTUM COMPUTATION

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Abstract. We demonstrate switching effects of a three-level atom placed within photonic crystals, which facilitates quantum computation in quantum bits (qubits). Here, we show two kinds of switching effects. One is the switching that can be used for measuring the quantum information of the qubit. The other is the switching that can be used for realizing the Hadamard transform of the qubit. These switching effects reveal that a quantum algorithm may be implemented with this photonic crystal system

1. Introduction

Recently, demand for a quantum bit [1] (qubit) to encode information for quantum computations has been increasing. Several approaches to the realization of a qubit have been proposed. For example, it has been reported that cold ions confined in a linear trap [2] and nuclear magnetic resonance [3] can be used as a qubit. Furthermore, a novel approach to realizing a qubit by controlling a photon has been investigated. In particular, attention has been focused on the photonic approach, where a qubit is realized by placing a single three-level atom within a periodic dielectric microstructure via the nanofabrication technique. [4]

The photonic approach is based on the photonic band gap (PBG) theory. The periodic dielectric microstructure shows the PBG. When the transition frequency of the three-level atom is near the edge of the PBG, classical light localization is formed. This leads to the storage of quantum information in populations on the two upper levels of the atom, which is relevant for optical memory effects of the qubit. [4] Recently, we found two kinds of switching effects in the atomic population on the upper level: one is mediated by changing the number of the localized modes in the emitted field [5,6], and the other is mediated by controlling the quantum interference [7].

In this paper, we demonstrate optical switching effects in a qubit composed of a three-level atom placed in a photonic crystal which exhibits the three-dimensional (3D) PBG. We find two optical switching effects for performing basic operations of the qubit. One is the optical switching for measuring the quantum information of the qubit. This is provided by switching both nonzero steady-state atomic populations on the two upper levels to zero population, which is mediated by changing the number of localized modes in the emitted field. The other is the optical switching for realizing the Hadamard transform of the qubit, which is the basis of quantum computing. This is provided by switching one of the nonzero steady-state atomic populations to zero population, which is mediated by controlling the quantum interference. Furthermore, we reveal the phases and strength of the external fields required for facilitating the two optical switching effects.

2. Qubit model

A qubit is required to form a coherent superposition of two or more measurable quantum states. When we use the atomic level $b_m(t)\,|m\rangle$ as the quantum state of the qubit, the quantum state can be measured as the spontaneous emission on the atomic transition from $|m\rangle$ to the ground level $|1\rangle$, which is characterized by the atomic population $n_m(t) \equiv |b_m(t)|^2$ and the transition frequency ω_{m1} between $|m\rangle$ and $|1\rangle$. We consider a three-level atom as the qubit composed of the ground level $|1\rangle$ and the additional two upper levels $|upper\rangle$:

$$|upper\rangle = b_3(t)|3\rangle + b_2(t)|2\rangle \, . \qquad (1)$$

Figure 1 shows a level diagram of this three-level atomic system in the Λ configuration, where the upper level $|3\rangle$ is dipole-coupled to the ground level $|1\rangle$ with a dipole moment d_{31}. δ is the detuning of the transition frequency ω_{31} from the upper band edge frequency ω_c, $\delta = \omega_{31} - \omega_c$.

Furthermore, the qubit is required to store quantum information in the quantum states. In the case of the qubit $|upper\rangle$, the quantum information is stored in the amplitude $b_k(t)$, (k=2, 3), where information is encoded by the value of the amplitude. It is required for achieving a nonzero value of the amplitude $b_k(t)$ to drive the atomic system. When the driven atom is in free space, the atomic population $n_k(t)$ on the upper level $|k\rangle$ exhibits simple exponential decay. Therefore, no quantum information is stored in the atomic population. On the other hand, when the driven atom is in the photonic crystal which exhibits a 3D PBG and the transition frequency is near the PBG, the atomic population $n_k(t)$ has the nonzero steady-state value $n_{ks} \equiv \lim_{t\to\infty} n_k(t)$, which corresponds to the storage of the quantum information in the qubit. Furthermore, we have found the exact solution of poles in the Laplace-transformed atomic population and revealed the switching effect in the steady-state atomic population on the upper level $|3\rangle$, n_{3s}. [5-7]

According to the above two requirements, we consider the two upper levels $|upper\rangle$ of the three-level atom placed within the photonic crystal as the qubit. Figure 2 shows the graphical representation of this qubit, which is the inverse opal system including spherical voids that have their internal surfaces coated by liquid-crystal molecules. In this system, three external fields are used. The pump pulse laser field of phase ϕ_p is used for preparing the initial atomic state in the form:

$$|\Psi(0)\rangle = \cos\theta|3\rangle + e^{i\phi_p}\sin\theta|2\rangle . \tag{2}$$

The phase θ of the superposition rate $\sin\theta$ characterizes the strength of the pump laser field. The control cw laser field of the phase ϕ_c and the Rabi frequency Ω is used for coupling the two upper levels of the atom, and the small DC field is used for controlling the detuning δ.

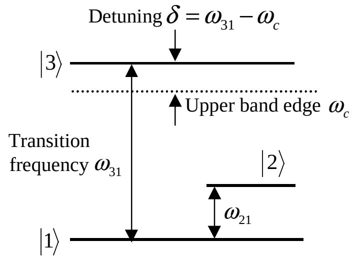

Figure 1. Level diagram of the three-level atomic system.

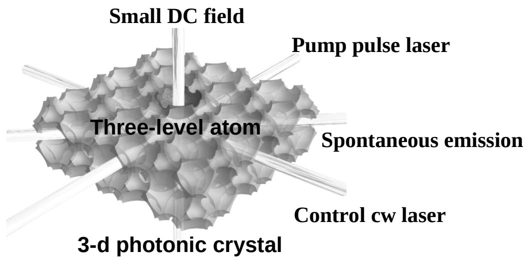

Figure 2. Graphical representation of the qubit

3. Quantum information of the qubit

The quantum information of the qubit $|upper\rangle$, which is encoded by the steady-state atomic population on the upper level $|k\rangle$, can be measured as spontaneous emission on the transition from $|k\rangle$ to $|1\rangle$. Furthermore, the spontaneous emissions from $|3\rangle$ and $|2\rangle$ are identified by their frequencies. Therefore, both information in the two upper states $b_3(t)|3\rangle$ and $b_2(t)|2\rangle$ can be measured simultaneously. It is required for measuring information in both $|3\rangle$ and $|2\rangle$ as their resultant spontaneous emissions to design a scheme for simultaneously switching both nonzero steady-state values of the atomic populations, n_{3s} and n_{2s}, to zero. This is relevant for the switching effect in both atomic populations, n_{3s} and n_{2s}.

We find that this switching effect is mediated by changing the number of localized modes in the emitted field. This is revealed by using the exact solution of poles [7], similar to the switching effect in the atomic population n_{3s} [5,6], which shows that the number of localized modes is characterized by the relation between the Rabi frequency Ω and the detuning δ. In $|\delta| < \Omega$, there is a single localized mode in the emitted fields, which leads to the formation of the nonzero steady states, n_{3s} and n_{2s}, on the upper levels $|3\rangle$ and $|2\rangle$, respectively. In $\delta > \Omega$, there is no localized mode in the emitted fields, which leads to the formation of zero steady states on the upper levels $|3\rangle$ and $|2\rangle$. In Fig. 3, we plot the value of the steady-state atomic population n_{ks} as a function of the Rabi frequency Ω and the detuning δ, which satisfies $|\delta| < \Omega$, for the phases ϕ $(=\phi_p - \phi_c)$ $=\pi$ and $\theta = \pi/4$. The plot region is represented by the shaded portion in Fig. 3(c). Figure 3 reveals that steady-state atomic populations n_{3s} and n_{2s} are nonzero or zero, depending on the Rabi frequency Ω and the detuning δ. Furthermore, the steady-state atomic populations n_{3s} and n_{2s} have the same cutoff frequency $\delta = \Omega$, which leads to the switching effect in both atomic populations.

This switching effect provides the optical scheme for measuring the quantum information of the qubit, where the quantum information in both quantum states $b_3(t)|3\rangle$ and $b_2(t)|2\rangle$ are measured as spontaneous emissions, simultaneously. The switching is realized by changing the Rabi frequency Ω and the detuning δ, which are controlled by the external fields in Fig. 2. Furthermore, the spontaneous emissions concerning $|3\rangle$ and $|2\rangle$ are characterized by the frequencies $\omega_{31} + \mathrm{Re}(u^2)$ and $\omega_{21} + \mathrm{Re}(u^2)$, where u [7] is the pole in the Laplace-transformed atomic population.

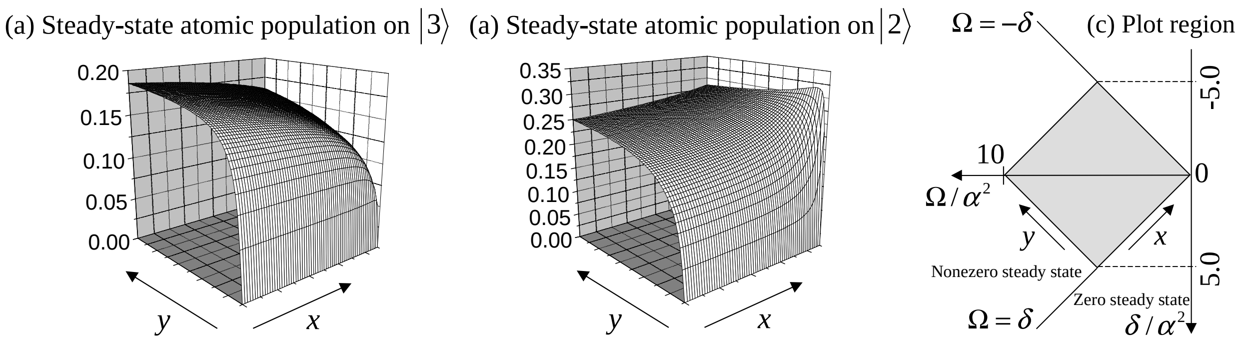

Figure 3. Steady-state atomic population as a function of the Rabi frequency and the detuning.

4. Hadamard transform of a qubit

The Hadamard transform is the basis of the quantum computation in the qubit, which transforms the quantum states from $(|3\rangle \pm |2\rangle)/\sqrt{2}$ to $|3\rangle$ or $|2\rangle$. In this section, we present a scheme for switching one of the nonzero values of the steady-state atomic populations, n_{3s} and n_{2s}, to zero. This is relevant for the switching effect in one of the atomic populations n_{3s} and n_{2s}, which may provide the basis of the Hadamard transform in the qubit.

We find that this switching effect is mediated by controlling the quantum interference between the upper states $|3\rangle$ and $|2\rangle$ in Eq. (1). The quantum interference can be controlled by the two phases ϕ and θ in Eq. (2). In the following, we consider the switching scheme for the case where one of the two phases is fixed.

First, we consider the case where the phase ϕ is fixed. The quantum interference between the upper states $|3\rangle$ and $|2\rangle$ leads to a periodical fluctuation in the steady-state atomic populations n_{3s} and n_{2s}, depending on $-\sin\phi$, where the maximum fluctuation is given by the phase $\phi = 3\pi/2$. For this phase value, we can form the atomic populations that have the zero steady state. In Fig. 4(a), we plot the value of the steady-state atomic population $n_k(t)$ as a function of the phase θ for the Rabi frequency $\Omega = 5\alpha^2$, the detuning $\delta = 0$ and the phase $\phi = 3\pi/2$ with

$$\alpha \approx \omega_{31}^{5/2} d_{31}^2 / (12\pi\varepsilon_0 \hbar c^3). \quad (3)$$

[4]

This shows that the steady-state atomic populations n_{3s} and n_{2s} are nonzero or zero, depending on the phase θ with a period π. Furthermore, the phases θ_{3s} and θ_{2s}, which give zero population for n_{3s} and n_{2s}, respectively, are different from each other.

Next, we consider the case where the phase θ is fixed. We find that the atomic populations, which have the zero steady state, can be formed by the phase θ_{3s} given by

$$\theta_{3s} = \pi/2 - \tan^{-1}\left[\left(u^2 - \delta\right)/\Omega\right]. \tag{4}$$

Here, the phase θ_{3s} is almost constant $\pi/4$ in $|\delta| < \Omega$. In Fig. 4(b), we plot the value of the steady-state atomic population n_{ks} as a function of the phase ϕ for the Rabi frequency $\Omega = 5\alpha^2$, the detuning $\delta = 0$ and the phase θ_{3s}. This shows that the steady-state atomic populations n_{3s} and n_{2s} are nonzero or zero, depending on the phase ϕ with a period 2π. Furthermore, the phases ϕ_{3s} and ϕ_{2s}, which give zero population for n_{3s} and n_{2s}, respectively, are different from each other.

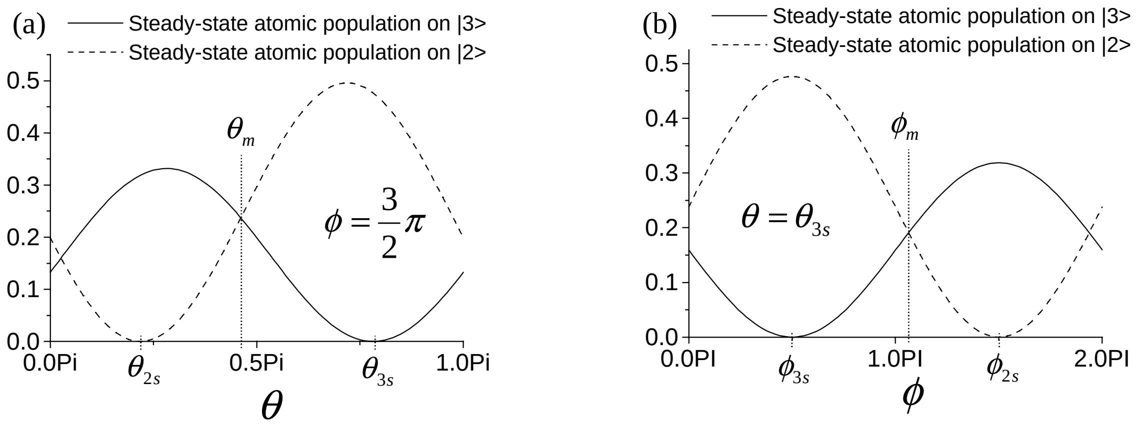

Figure 4. Steady-state atomic population as a function of the phases.

In Figs. 4(a) and 4(b), we plot the phases θ_m and ϕ_m, which give $n_{3s} = n_{2s}$, respectively. Using the phases in Fig. 4, we find that the switching effect in one of the atomic populations is realized by changing the pair of phases (θ, ϕ) from $(\theta_m, 3\pi/2)$ to $(\theta_{ks}, 3\pi/2)$ or (θ_{3s}, ϕ_m) to (θ_{3s}, ϕ_{ks}), which provides the optical switching scheme for realizing the Hadamard transform of the qubit.

5. Conclusion

We have demonstrated the optical switching in a qubit composed of a three-level atom placed in a photonic crystal. First, we found the optical switching effect for measuring the quantum information, which is provided by switching both nonzero steady-state atomic populations on the two upper levels to zero population, and its cutoff frequency. Next, we found the optical switching for performing the Hadamard transform, which is provided by switching one of the nonzero steady-state atomic populations to zero population, and the phase values required for this optical switching. These two optical switching effects present a possibility of using the photonic crystal for a qubit.

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