

# PARALLEL TRANSMISSION AND SPATIAL FILTERING IN A GRADED-INDEX OPTICAL PLANAR WAVEGUIDE

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**Abstract** The main purpose of the present studies is to analyze the transmission and image distortion characteristics using graded-index optical planar waveguides. For the studies on transmission characteristics, we considered a model with Ti-diffused  $\text{LiNbO}_3$  waveguides. By theoretical calculations in the space domain, we derived the point spread function (PSF) when a spatial impulse is used as input. For the electric field distribution inside the waveguide, we assumed Hermite-Gaussian functions and derived the first order perturbation solution, and performed the theoretical calculations. We evaluated the propagation characteristics of an optimized GRIN type optical waveguide with respect to propagation distance. The periodic focusing points in the propagation direction were confirmed. In order to remove the noise component during the transmission, we considered spatial filters. The image transmission and processing using spatial filters are discussed in this paper.

## 1. Introduction

Integrated optical waveguide type functional devices capable of handling image transmission and parallel data have become indispensable for an optical computer[1]. Graded-index optical waveguides play an important role in the realization of devices for image transmission and parallel data handling. Graded-index multimoded optical fibers have so far been used for 2-D image transmission[2]. In a similar way, it has been reported that graded-index type thin film optical waveguide with Gaussian index distribution in the transverse direction exhibit focusing properties and can be used for 1-D image transmission[3].

The main purpose of the present studies is to analyze the transmission characteristics using graded index waveguides and to reduce image distortion. For the studies on transmission characteristics, we considered a model with Ti-diffused  $\text{LiNbO}_3$  waveguides. By theoretical calculations in the space

domain, we derived the point spread function (PSF) when a spatial impulse is used as input[4]. For the electric field distribution inside the waveguide, we assumed Hermite-Gaussian functions and derived the first order perturbation solution, and performed the theoretical calculations.

## 2. Analysis of Parallel and Image Transmission

Fig.1 shows the transmission device model considered for the analysis. For the studies on transmission characteristics, we considered a model with Ti-diffused  $\text{LiNbO}_3$  waveguides[5].

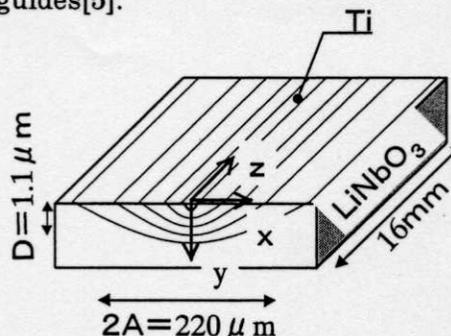


Fig.1 Model of image transmission device

Graded-index type thin film optical waveguide with Gaussian index distribution in the transverse direction exhibit focusing properties and can be used for 1-D image transmission, as shown in Fig.2.

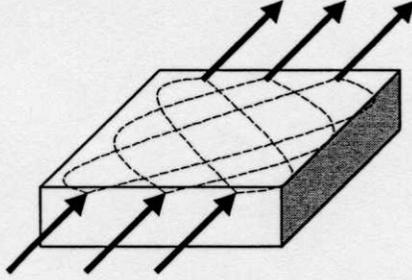


Fig.2 Transmission routes of signals

The image transmission characteristics such as resolution, distortion and imaging can be evaluated by means of the field distribution and propagation constants of the graded-index waveguide. Thus, in this analysis we considered the transverse electric field and its propagation constants obtained from the following wave equation (1).

$$\left\{ \nabla^2 + k^2 N^2(x, y) - \beta^2 \right\} \phi(x, y) = 0 \quad (1)$$

where

$$N^2(x, y) = Nb^2 + (Ns^2 - Nb^2)$$

$$\times \left[ 1 - \left( \frac{x}{A} \right)^2 + h \left( \frac{x}{A} \right)^4 \right] \exp \left[ - \left( \frac{y}{D} \right)^2 \right] \quad (2)$$

and  $k$  is wave number.  $Nb$  and  $Ns$  are the index of refraction of the substrate and the middle of the core.  $A$  and  $D$  are the width and depth of the waveguide where the index of refraction becomes 1/e of its value at  $x=0, y=0$ .  $h$  is a parameter governing the index distribution in the lateral direction. This parameter can be controlled in the thermal diffusion process.

For the present case of an index distribution, the electric field is given by

$$\phi_{nm}^{(0)}(x, y) = \phi_n(x) \phi_m(y) \quad (3)$$

where each  $\phi$  is given by Hermite-Gaussian functions as

$$\phi_i(\xi_j) = C_i H_i(\xi_j) \exp \left( - \frac{\xi_j^2}{2} \right), \quad \begin{matrix} i = n, m \\ j = x, y \end{matrix} \quad (4)$$

where

$$\xi_x = \frac{\alpha_x^{1/2} x}{A}, \quad \xi_y = \frac{\alpha_y^{1/2} y}{D}, \quad \Delta = \frac{Ns^2 - Nb^2}{Ns^2}$$

$$C_n = \frac{(\alpha_x / \pi)^{1/4}}{(2^n n! A)^{1/2}}, \quad C_m = \frac{(\sqrt{2} \alpha_y / \pi)^{1/4}}{(2^l l! D)^{1/2}}$$

$$\alpha_x = Ak(2\Delta)^{1/2} Ns, \quad \alpha_y = Dk(2\Delta)^{1/2} Ns$$

and  $H_i$  are Hermite polynomials.  $n, m$  are the order of propagation modes in the  $x$  and  $y$  direction.  $l = 2m + 1$ . The propagation constants for this case are given by

$$\beta_{nm}^{(0)2} = k^2 Ns^2 - (2n + 1) \frac{\alpha_x}{A^2} - (2l + 1) \frac{\alpha_y}{D^2} \quad (5)$$

The reason of choosing odd modes in the  $y$ -axis direction is that, as a consequence of the abrupt index discontinuity at the waveguide surface, only those modes that are antisymmetric with respect to the plane  $x=0$  exist[6].

Then, applying the perturbation method, the first-order perturbation correction to the field and propagation constants is given by

$$\phi_{nm}^{(1)}(x, y) = \sum_p \sum_q a_{pq} \phi_{pq}^{(0)}(x, y), \quad \begin{matrix} p \neq n \\ q \neq m \end{matrix} \quad (6)$$

where

$$a_{pq} = \frac{\iint \phi_{pq}^{(0)}(x, y) H' \phi_{pq}^{(0)}(x, y) dx dy}{\beta_{nm}^{(0)2} - \beta_{pq}^{(0)2}} \quad (7)$$

$$\beta_{nm}^{(1)2} = \iint \phi_{nm}^{(0)}(x, y) H' \phi_{nm}^{(0)}(x, y) dx dy \quad (8)$$

where  $H'$  corresponds to those terms of the index distribution of order greater than four. Finally, the total electric field and propagation constants are given by

$$\phi_{nm}(x, y) = \phi_{nm}^{(0)}(x, y) + \phi_{nm}^{(1)}(x, y) \quad (9)$$

$$\beta_{nm}^2 = \beta_{nm}^{(0)2} + \beta_{nm}^{(1)2} \quad (10)$$

The image transmission characteristics are analyzed by means of the impulse response of the waveguide, since any arbitrary input can be regarded as a linear combination of shifted point sources.

Then, the electric field distribution can be expressed by the superposition of  $\phi_{nm}$  as

$$E(x, y, z) = \sum_{n=0}^N \sum_{m=0}^M C_{nm} \phi_{nm}(x, y) \exp(-jz\beta_{nm}) \quad (11)$$

For a spatial impulse input, expressed by

$E = \delta(x - x_i) \delta(y - y_i)$ , the coefficients  $C_{nm}$  are obtained. Therefore, the PSF of the waveguide is given by

$$h(x, y, z) = \sum_{n=0}^N \sum_{m=0}^M \frac{\phi_{nm}(x_i, y_i)}{\iint |\phi_{nm}(x, y)|^2 dx dy} \phi_{nm}(x, y) \exp(-jz\beta_{nm}) \quad (12)$$

### 3. Numerical Results

The impulse response is simulated using equation (12) and the following parameters.  $A=110 \mu\text{m}$ ,  $D=1.1 \mu\text{m}$ ,  $N_s=2.296$ ,  $N_b=2.286$ ,  $h=0.25$  and a He-Ne laser with a wavelength of  $\lambda=0.6328 \mu\text{m}$ .

For the case of on-axis propagation, the electric field distribution of a point image along the direction of propagation is illustrated in fig.3. This figure shows that the amplitude of the image at the focusing points decreases as the propagating distance increases.

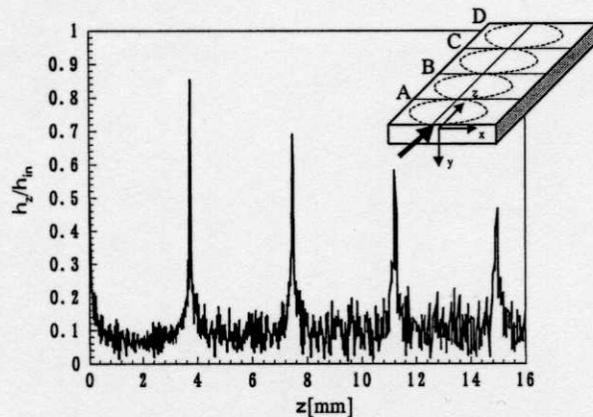


Fig.3 The electric field variation along the transmission direction

The field distributions at the second and fourth focusing points are plotted in Fig.4 and Fig.5 respectively. A comparison of these figures shows that the point image spreads across the waveguide and lateral lobes starts appear as the distance increases. Lateral lobes become noise for image transmission.

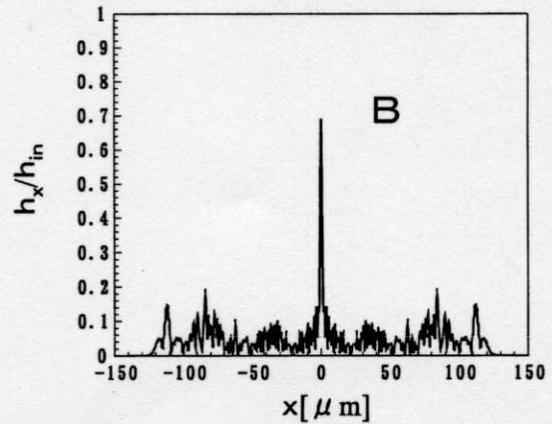


Fig.4 The electric field at B point on z-axis

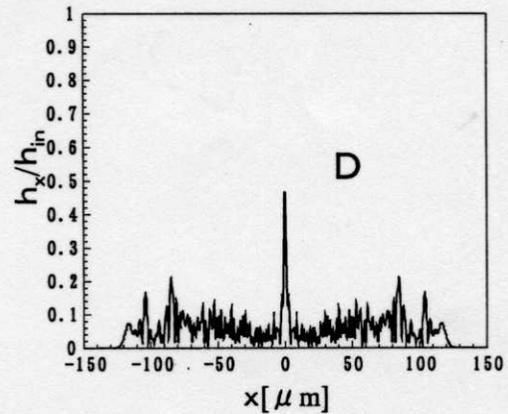


Fig.5 The electric field at D point on z-axis

### 4. Spatial filter for image reconstruction

The output image  $g_o(x)$  is given by

$$g_o(x) = \int g_i(x') h(x - x') dx' \quad (13)$$

where  $g_i(x)$  is the input image. By Fourier transforming equation (13) the spatial frequency spectrum of the image is written as

$$G_o(u_x) = H(u_x) G_i(u_x) \quad (14)$$

By means of a filter with a proper transfer function, the output image can be transformed to the one similar to the input. Then regarding the Inverse filter theory[7], the filter

transmittance  $M(u_x)$  is given by

$$M(u_x) = \frac{H^*(u_x)}{|H(u_x)|^2} \quad (15)$$

Fig.6 shows the input image. Fig.7 and Fig. 8

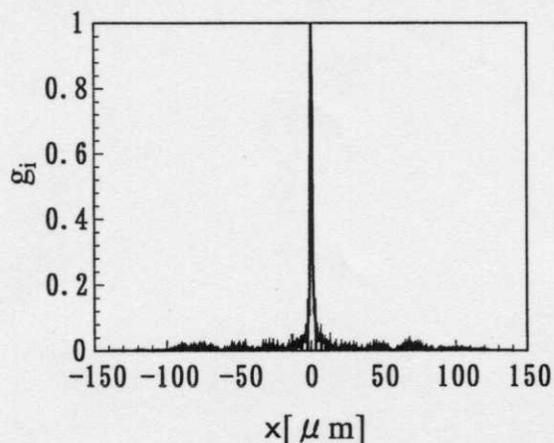


Fig.6 Input image

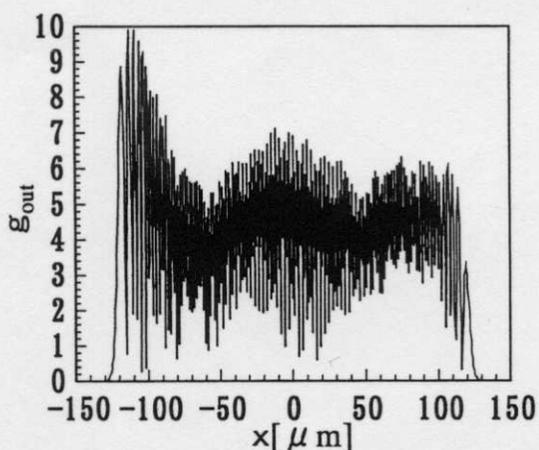


Fig.7 Output image without filter

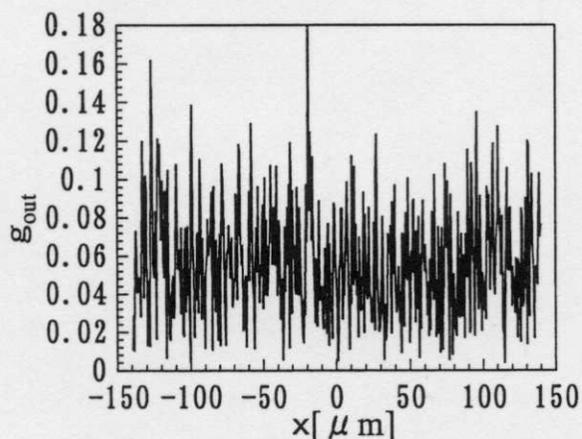


Fig.8 Output image after passing through filter

show the output image without filter and after passing through filter.

Electric fields are suppressed and the input image is retrieved to some extent. Since the image is not completely retrieved, we plan to investigate various other filters.

## 5. Conclusion

The image transmission characteristics of an optical graded-index waveguide have been analyzed. And we proposed a spatial filter. The input image is retrieved to some extent through the filter. Since the image is not completely retrieved, we plan to investigate various other filters.

## References

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