Fabrication of three-dimensional photonic crystals by holographic lithography

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1.Introduction

Recently, photonic crystals have been given considerable attention because it is expected that they might give a significant breakthrough in many photonic applications. However, it is rather hard to fabricate the three-dimensional photonic crystal. Many fabrication methods have been proposed and studied. In these fabrication methods, we take notice of the holographic interference method as an elegant way to fabricate dielectric photonic crystals[1).

In this paper we describe the relations between incident directions of four interference waves and the resultant interference fringe in the holographic interference method based on the concept of crystal lattice vector and reciprocal lattice vector in the solid state physics.

2.Theoretical analysis

Light intensity distribution for four-wave interference is given by extending the two-wave interference equation[2) as,

$$\boldsymbol{I} = \sum_{m=0}^{3}\sum_{n=0}^{3}\boldsymbol{U}_m\boldsymbol{U}_n\cos\{(\boldsymbol{K}_m-\boldsymbol{K}_n)\cdot\boldsymbol{r}\}, \tag{1}$$

where $\boldsymbol{U}_m$ and $\boldsymbol{U}_n$ are the amplitudes of the incident waves, $\boldsymbol{K}_m$ and $\boldsymbol{K}_n$ are their wave number vectors and $\boldsymbol{r}$ is the position vector that represents observation point, that is, interference point. When a point $\boldsymbol{r}$ satisfies the following three equations, the interference fringe intensity becomes maximum at the point.

$$(\boldsymbol{K}_i-\boldsymbol{K}_0)\cdot r = 2\pi\, l_i \quad \text{for} \quad i=1\sim 3, \tag{2}$$

where l_i is an integer. The sum of three equations in Eq.(2) is written as,

$$(\Delta\boldsymbol{K}_{10}+\Delta\boldsymbol{K}_{20}+\Delta\boldsymbol{K}_{30})\cdot\boldsymbol{r} = 2\pi\boldsymbol{L}, \tag{3}$$

where

$$\Delta\boldsymbol{K}_{i0} = \boldsymbol{K}_i-\boldsymbol{K}_0 \quad \text{for} \quad i=1\sim 3, \tag{4}$$

and

$$\boldsymbol{L} = l_1+l_2+l_3, \tag{5}$$

Here we consider the crystal lattice vector ρ and reciprocal lattice vector $\boldsymbol{G}$. They are defined respectively as[3),

$$\rho = m\boldsymbol{a}+n\boldsymbol{b}+p\boldsymbol{c}, \tag{7}$$

$$\boldsymbol{G} = h\boldsymbol{A}+k\boldsymbol{B}+l\boldsymbol{C}, \tag{6}$$

where m, n, p, h, k and l are integers and $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are the primitive translation vectors of the crystal lattice and $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are the primitive translation vectors of the reciprocal lattice. The primitive translation vectors of the crystal lattice and of the reciprocal lattice are connected as,

$$\boldsymbol{A} = 2\pi\frac{\boldsymbol{b}\times\boldsymbol{c}}{\boldsymbol{a}\cdot\boldsymbol{b}\times\boldsymbol{c}},\ \boldsymbol{B} = 2\pi\frac{\boldsymbol{c}\times\boldsymbol{a}}{\boldsymbol{a}\cdot\boldsymbol{b}\times\boldsymbol{c}},\ \boldsymbol{C} = 2\pi\frac{\boldsymbol{b}\times\boldsymbol{a}}{\boldsymbol{a}\cdot\boldsymbol{b}\times\boldsymbol{c}}. \tag{8}$$

The inner product between $\boldsymbol{G}$ and ρ becomes from Eq.(8) as,

$$\boldsymbol{G}\cdot\rho = 2\pi\,\boldsymbol{L}', \tag{9}$$

where $\boldsymbol{L}' = hm+kn+lp$, that is, $\boldsymbol{L}'$ is an integer.

To compare the Eq.(3) and Eq.(9), we assume that the sum of three equations $\Delta\boldsymbol{K}_{10}+\Delta\boldsymbol{K}_{20}+\Delta\boldsymbol{K}_{30}$ in Eq.(3) agree with the reciprocal lattice vector $\boldsymbol{G}'$. Then the vectors $\Delta\boldsymbol{K}_{i0}$ ($i = 1\sim 3$) can be expressed by using a linear combination of the proper primitive translation vectors $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ in the reciprocal lattice space as,

$$\Delta\boldsymbol{K}_{i0} = h_i\boldsymbol{A}+k_i\boldsymbol{B}+l_i\boldsymbol{C} \quad \text{for} \quad i=1\sim 3, \tag{10}$$

where h_i, k_i and l_i are integers. When three equations in Eq.(10) are substituted for Eq.(3), then we get

$$(h'\boldsymbol{A}+k'\boldsymbol{B}+l'\boldsymbol{C})\cdot r = 2\pi\boldsymbol{L}, \tag{11}$$

where $h'=h_1+h_2+h_3$, $k'=k_1+k_2+k_3$ and $l'=l_1+l_2+l_3$. Further, we can rewrite Eq.(11) by using a vector $\boldsymbol{G}'$ as,

$$\boldsymbol{G}' \cdot \boldsymbol{r} = 2\pi\ \boldsymbol{L}, \tag{12}$$

where $\boldsymbol{G}'=h'\boldsymbol{A}+k'\boldsymbol{B}+l'\boldsymbol{C}$. Eq.(12) is equivalent to Eq.(9). This means that position vector $\boldsymbol{r}$ corresponds to crystal lattice vector ρ if the vector $\boldsymbol{G}'$ is corresponds to reciprocal lattice vector $\boldsymbol{G}$. As is obvious from this discussion, if we determine incident angles and directions of interference beams so that the $\Delta\boldsymbol{K}_{10}$, $\Delta\boldsymbol{K}_{20}$ and $\Delta\boldsymbol{K}_{30}$ satisfy the reciprocal lattice condition for the desired crystal, that is, Eq.(10), the interference fringe intensity becomes maximum at the corresponding crystal lattice point. As a result, the desired photonic crystal structure can be fabricated by recording the interference fringe in the photosensitive materials such as UV curing resin.

3.Numerical results

To verify the above theory, it is applied for the face-centered cubic (f.c.c.) lattice. As the simplest example, the following values were chosen for Eq.(10).

$$\begin{aligned} &\Delta \boldsymbol{K}_{10} = \boldsymbol{A}, \text{ that is, } h_1 = 1, k_1 = 0, l_1 = 0;\\ &\Delta \boldsymbol{K}_{20} = \boldsymbol{B}, \text{ that is, } h_2 = 0, k_2 = 1, l_2 = 0;\\ &\Delta \boldsymbol{K}_{30} = \boldsymbol{C}, \text{ that is, } h_3 = 0, k_3 = 0, l_3 = 1, \end{aligned} \tag{13}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are the primitive translation vectors of the simplest f.c.c as,

$$\begin{aligned} \boldsymbol{A} &= 2\pi\ / d\,[\ -1,\ \ 1,\ \ 1],\\ \boldsymbol{B} &= 2\pi\ / d\,[\ \ 1, -1,\ \ 1],\\ \boldsymbol{C} &= 2\pi\ / d\,[\ \ 1,\ \ 1, -1]. \end{aligned} \tag{14}$$

We can calculate $\boldsymbol{K}_i$ (i = 0～3) from Eq.(4) and Eq.(13) as,

$$\begin{aligned} \boldsymbol{K}_0 &= 2\pi\ / d\,[\ -3/2, -3/2, -3/2\,],\\ \boldsymbol{K}_1 &= 2\pi\ / d\,[\ -5/2, -1/2, -1/2\ ],\\ \boldsymbol{K}_2 &= 2\pi\ / d\,[\ -1/2, -5/2, -1/2\ ],\\ \boldsymbol{K}_3 &= 2\pi\ / d\,[\ -1/2, -1/2, -5/2\ ], \end{aligned} \tag{15}$$

where d is the lattice constant and calculated as $d=27^{1/2}\ \lambda/2$. For the recording wavelength of 441.6nm (Hd-Cd laser), the lattice constant d is 1.1473 μm. Figure 1 shows the appearance of four wave vectors of Eq.(15). The three vectors $\boldsymbol{K}_1$, $\boldsymbol{K}_2$ and $\boldsymbol{K}_3$ are symmetrically arranged around the central vector $\boldsymbol{K}_0$. Figure 2 shows cross sectional views of the light intensity distribution on each plane, and Fig.3 shows three-dimensional view of

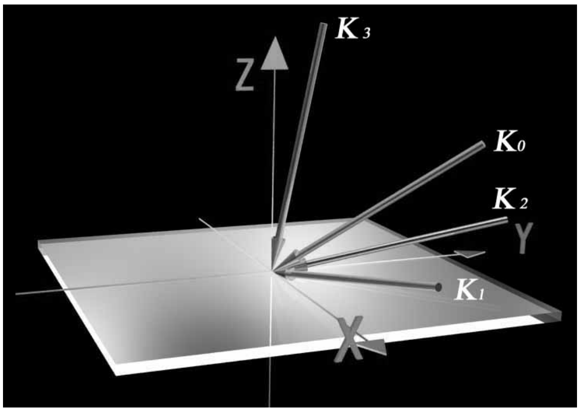
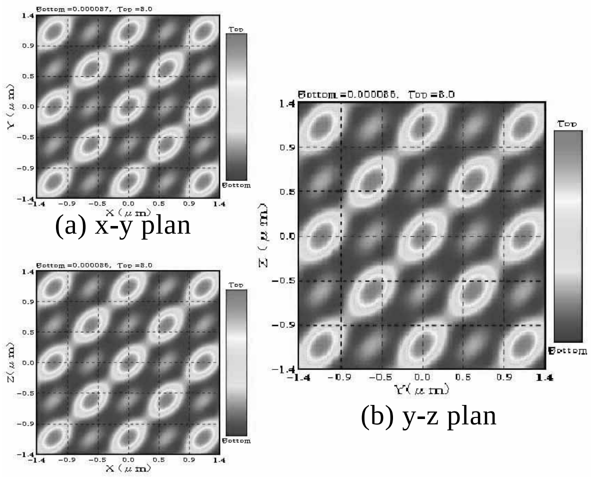


Fig.1 The appearance of four wave vectors of Eq.(15) that create f.c.c inference fringe pattern.



(a) x-y plan

(b) y-z plan

(c) z-x plan

Fig.2 Light intensity distribution on each cross-section. (a) x-y plan, (b) y-z plan, (c) z-x plan. The length of each side is 2.8 μm.

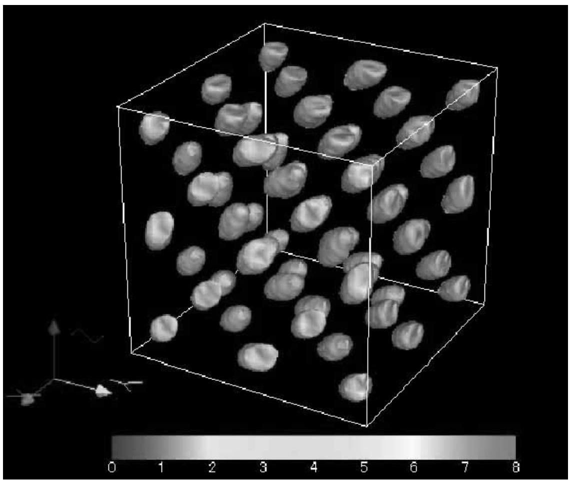


Fig.3 Three-dimensional display of the light intensity distribution for the arrangement shown in Fig.1. Only parts that have intensity greater than 50% of the maximum value are displayed. The length of each edge is 2.8μm.

the light intensity. In Fig.3, only parts that have intensity greater than the predetermined value is displayed with considering the recording process that may have a recording threshold value.

Next we can choose the other pattern to create the simplest f.c.c interference pattern,

$$\begin{aligned} \Delta \boldsymbol{K}_{13} &= \boldsymbol{A}, \\ \Delta \boldsymbol{K}_{02} &= \boldsymbol{B}, \\ \Delta \boldsymbol{K}_{21} &= \boldsymbol{C}, \end{aligned} \tag{16}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are the same values as those in Eq.(14). We can determine the wave vectors in the same way as the previous discussion as,

$$\begin{aligned} \boldsymbol{K}_0 &= 2\pi / d\,[\ 1,\ 0,\ 1/2\ ], \\ \boldsymbol{K}_1 &= 2\pi / d\,[\ -1,\ 0,\ 1/2\ ], \\ \boldsymbol{K}_2 &= 2\pi / d\,[\ 0,\ 1,\ -1/2\ ], \\ \boldsymbol{K}_3 &= 2\pi / d\,[\ 0,\ -1,\ -1/2\ ], \end{aligned} \tag{17}$$

where d is the lattice constant and calculated as $d=5^{1/2}\ \lambda/2$ (0.494 μm for wavelength $\lambda=$ 441.6nm). Figure 4 shows the appearance of four wave vectors of Eq.(17). The two beams are incident from the top and the rest two beams are incident from the bottom, and these are symmetrically arranged each other. Figure 5 shows three-dimensional view of the light intensity.

The other crystal structure can be produced by this method, for example the body-centered cubic (b.c.c.) lattice. We choose the following values for Eq.(10).

$$\begin{aligned} \Delta \boldsymbol{K}_{10} &= \boldsymbol{A}, \text{ that is, } h_1=1, k_1=0, l_1=0; \\ \Delta \boldsymbol{K}_{20} &= \boldsymbol{B}, \text{ that is, } h_2=0, k_2=1, l_2=0; \\ \Delta \boldsymbol{K}_{30} &= \boldsymbol{C}, \text{ that is, } h_3=0, k_3=0, l_3=1, \end{aligned} \tag{18}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are the primitive translation vectors of the simplest b.c.c. as,

$$\begin{aligned} \boldsymbol{A} &= 2\pi / d\,[\ 1,\ 1,\ 0], \\ \boldsymbol{B} &= 2\pi / d\,[\ 0,\ 1,\ 1], \\ \boldsymbol{C} &= 2\pi / d\,[\ 1,\ 0,\ 1]. \end{aligned} \tag{19}$$

We can calculate $\boldsymbol{K}_i$ (i = 0～3) from Eq.(4) and Eq.(18) as,

$$\begin{aligned} \boldsymbol{K}_0 &= 2\pi / d\,[\ -1/2,\ -1/2,\ -1/2\ ], \\ \boldsymbol{K}_1 &= 2\pi / d\,[\ 1/2,\ 1/2,\ -1/2\ ], \\ \boldsymbol{K}_2 &= 2\pi / d\,[\ -1/2,\ 1/2,\ 1/2\ ], \\ \boldsymbol{K}_3 &= 2\pi / d\,[\ 1/2,\ -1/2,\ 1/2\ ], \end{aligned} \tag{20}$$

where d is the lattice constant and calculated as $d=3^{1/2}\ \lambda/2$ (0.382 μm for wavelength $\lambda=$ 441.6nm). Figure 6 shows the appearance of four wave vectors of Eq.(20). The four beams are symmetrically arranged each other similarly as Fig.4, but the incident angles are different. Figure.7 shows three-dimensional view of the light intensity.

From these results it is verified that the desired crystal structure in the solid state physics can be fabricated according to the theory developed above.

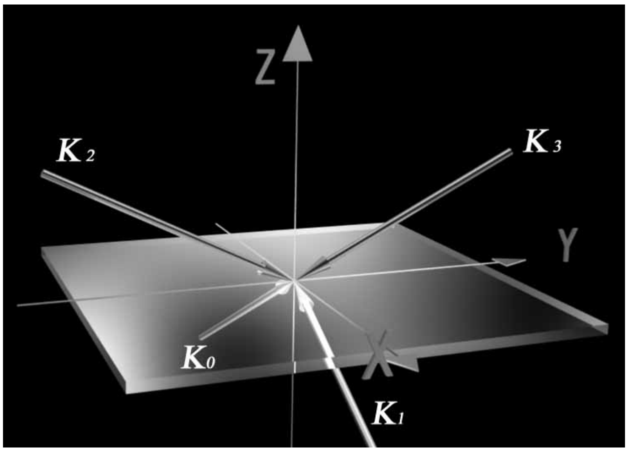


Fig.4 The appearance of four wave vectors of Eq.(17) that create f.c.c. inference fringe pattern. The two beams are incident from the top and the rest two beams are incident from the bottom, and these are symmetrically arranged each other.

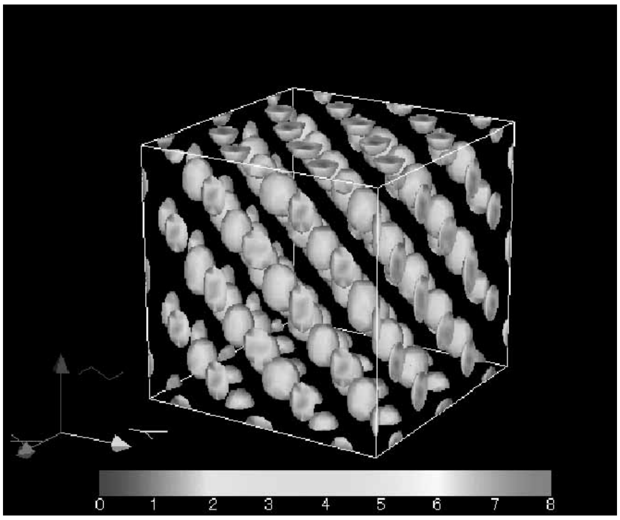


Fig.5 Three-dimensional display of the light intensity distribution for the arrangement shown in Fig.4. Only parts that have intensity greater than the 50% of the maximum value are displayed. The length of each edge is 1.4 μm. Compared with Fig.3, the shape of cell that construct this f.c.c lattice is closer to a sphere.

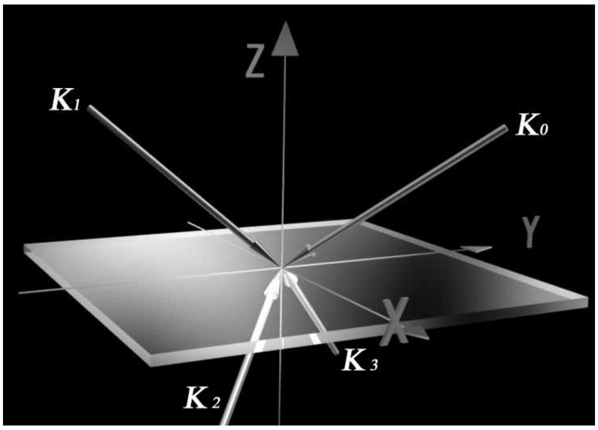


Fig.6 The appearance of four wave vectors of Eq.(20) that create b.c.c. inference fringe pattern. The four beams are symmetrically arranged each other similarly as Fig.4, but the incident angles are different.

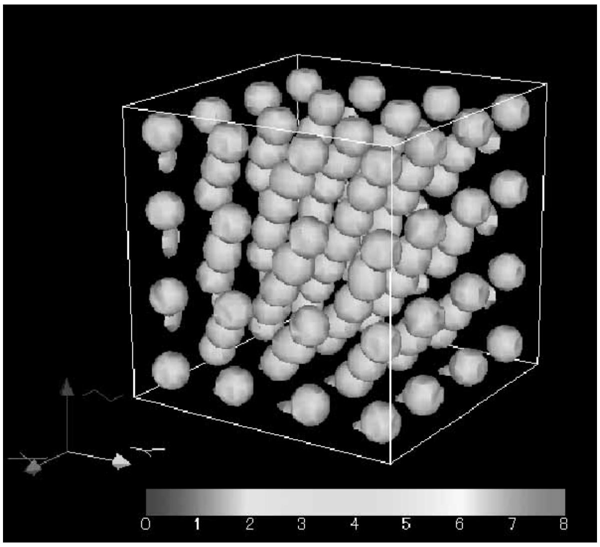


Fig.7 Three-dimensional display of the light intensity distribution for the arrangement shown in Fig.6. Only parts that have intensity greater than the 50%of the maximum value are displayed. The length of each edge is 1.4 μm.

4.Conclusion

As an example, four-wave interference has been theoretically and numerically analyzed. The derived theory, however, gives a general rule to determine the incident condition for interference beams in fabricating photonic crystals by holographic lithography method.

References

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