Refractive Index Profile Determination from Mode Indices Spectra using Iterative Characteristic Matrix Method

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Abstract

The reconstruction of refractive index profile of planar waveguides from its mode indices spectra using iterative application of Characteristic Matrix mode solver under the control of non-linear least squares optimization algorithm is described. The applications of the method for various waveguides, either step or graded index, either monotonically decreasing from surface or not were demonstrated. The possibility to reconstruct refractive index profile without knowing the profile function expression in advance was also demonstrated.

1. Introduction

Knowing the refractive index profile is of great importance in characterizing optical waveguides. By measuring the refractive index profile of various samples produced by particular fabrication technology, study that leads to the modeling of the fabrication process can be carried out. Moreover, refractive index profile is related to many important features like bandwidth, mode profile, and coupling efficiency of waveguides. Therefore, index profiling can be used to find out optimal waveguide structures for certain applications. Among many methods [1-5] that are used to determine the refractive index profile of optical waveguides, reconstruction of refractive index profile by processing its measured mode indices is the most widely used one, since the measurement can be carried out by simple m-line set-up. Generally, the refractive index profile is reconstructed from the mode indices by inversing the WKB eigen-value equation[1,3]. However, this method is applicable only to waveguides with index profile monotonically decreasing from the surface.

In this paper we describe an alternative method to reconstruct the refractive index profile of planar waveguides from its measured mode effective indices by applying the characteristic matrix mode solver[6] iteratively under the framework of least squares optimization algorithm until profile that has best fitted mode indices spectra is obtained. The scheme of the proposed method is almost similar to those of Shiozawa[4] and Caccavale[5], but we use different optimization algorithm, different mode solver, and demonstrate the possibility to reconstruct the refractive index profile without knowing the profile function expression in advance. The method can be applied to various index profile, including those that has sharp changes and not monotonically decreasing from the surface. Since it uses data fitting method, it is relatively insensitive to noises caused by measurement errors. As it employs mode solver, this method can be easily extended to measurement carried out under various environments, like m-line measurement using multiple wavelengths[3], both polarizations[3], various external over-claddings, various intensities, etc. since parameters like dispersion, birefringence, and non-linearity can be incorporated into the mode solver without much escalating its complexity. Thus, it is applicable to either single or multi-mode waveguides.

2. Method

Firstly, a set of mode effective indices of the waveguide are measured using prism-coupler. This measurement can be carried out for both polarizations, and if necessary at multiple wavelengths, or other variations in measurement parameters in order to obtain sufficient number of measured data. A set of index profile parameters to be optimized are then defined. If the analytic expression of the refractive index profile is known in advance, these parameters usually consist of waveguide depth, maximum index variation, and shape parameter for graded index waveguide; or number of layer, thickness and refractive index of layers for step index waveguide. If the analytic expression of the refractive index profile is not known, then the refractive indices and thickness of layers in multilayer structure will be adopted as the parameters to be optimized. Initial guess and bounds of the parameters are then generated from the measured effective indices and knowledge on typical value of corresponding parameters. These parameters and the measured mode indices spectra are fed into non-linear least-squares regression algorithm formulated according to the Trust-region Reflective-Newton method. The optimization of the parameters are then carried out iteratively until the calculated mode indices spectra best fits with the measured data as indicated by minimum value of squared of two-norm of error expressed by

$$\|\varepsilon\|_2^2 = \sum_{con=con1}^{conM} \sum_{m=0}^{N-1} \left[n_{effcalc}(m, con, p_1, .., p_k) - n_{effmea}(m, con) \right]^2 \qquad (1)$$

where $n_{effcalc}$ $(m,con,p_1,\ldots,p_k)$ is the calculated m^{th} order mode effective index at measurement condition con (wavelength, polarization, etc.) with profile parameters p_1 until p_k, and $n_{effmea}(m,con)$ is the corresponding measured mode indices, while N is the number of modes under the corresponding measurement circumstances. At each step of the iterations, the trial parameter values are used to generate a multilayer approximation to the waveguide and calculate its mode indices spectra under the corresponding measurement condition by means of Characteristic Matrix Method. This approximation is carried out in a non-uniform slicing manner, hence area with larger index gradient will be represented by more layers as depicted in Fig. 1. As this method relies on iterative optimization algorithm and there is not unique index profile that can produce the same mode index spectra, this method requires carefully selected initial values. As the highest index of the waveguide is generally close to the 0^{th} order mode index, then this mode index value can be used to generate the initial guess to the index difference Δ. The initial value of the thickness or depth of the guiding region is also determined by using the information on the 0^{th} order mode index value and the number of modes through simple rules of thumb. With proper selection on initial values along with upper and lower bounds restriction of the parameters, this method will converge gracefully.

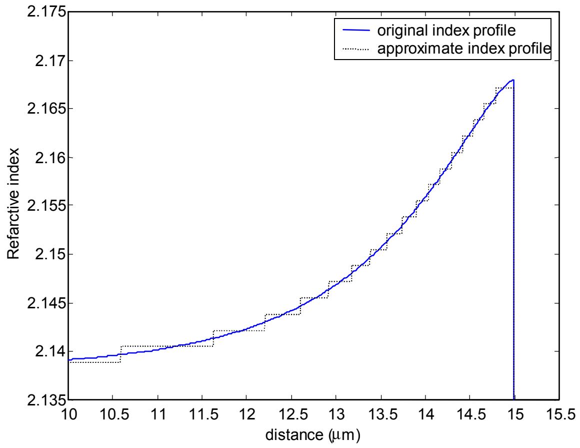

Figure 1. Approximation of graded index waveguide with a multilayer structure.

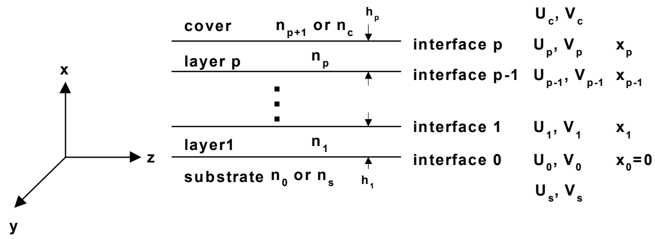

Figure 2. Multilayer structure model.

The multilayer structure model used for the calculation is shown in Fig. 2. According to the characteristic matrix method[6,7], the field at the i^{th} boundary can be expressed as a transfer of the field at 0^{th} boundary (substrate-first layer) by the characteristic matrix of multilayer stack in between $\mathsf{M}i$, namely

$$\begin{bmatrix} U_i \\ V_i \end{bmatrix} = \mathsf{M}_i \begin{bmatrix} U_0 \\ V_0 \end{bmatrix} \quad (2)$$

where $U=E_y$, $V=\omega\mu H_z$ for TE polarization, and $U=H_y$ and $V=\omega\varepsilon_0 E_z$ for TM polarization, while

$$\mathsf{M}_i = M_i \ldots M_1 = \begin{bmatrix} \mathsf{m}_{i_{11}} & \mathsf{m}_{i_{12}} \\ \mathsf{m}_{i_{21}} & \mathsf{m}_{i_{22}} \end{bmatrix} \quad (3)$$

with characteristic matrix of i^{th} layer M_i is given by

$$M_i = \begin{bmatrix} \cos(\kappa_i h_i) & -j\sin(\kappa_i h_i)/\kappa_i \\ -j\kappa_i \sin(\kappa_i h_i) & \cos(\kappa_i h_i) \end{bmatrix} \quad (4)$$

and

$$M_i = \begin{bmatrix} \cos(\kappa_i h_i) & j\, n_i^2 \sin(\kappa_i h_i)/\kappa_i \\ j\kappa_i \sin(\kappa_i h_i)/n_i^2 & \cos(\kappa_i h_i) \end{bmatrix} \quad (5)$$

for TE and TM polarization, respectively. In these expressions, $\kappa_i^2=k_0^2 n_i^2-k_0^2 n_{eff}^2$, $k_0=2\pi/\lambda_0$, while n_i is the refractive index of layer i, and h_i is the thickness of corresponding layer. For multilayer planar waveguide with p+2 layers, the effective indices of guided mode (n_{eff}) can be obtained by applying standard numerical root finding method to the dispersion equations:

$$j(\gamma_s \mathsf{m}_{p_{22}} + \gamma_c \mathsf{m}_{p_{11}}) = \gamma_s \gamma_c \mathsf{m}_{p_{12}} - \mathsf{m}_{p_{21}} \quad (6)$$

and

$$-j\left(\frac{m_{p22}\gamma_s}{n_s^2}+\frac{m_{p11}\gamma_c}{n_c^2}\right)=\frac{\gamma_s\gamma_c m_{p12}}{n_s^2 n_c^2}-m_{p21} \qquad (7)$$

for TE and TM polarization, respectively. In these expressions $\gamma_s^2=k_0^2n_{eff}^2-k_0^2n_s^2$, $\gamma_c^2=k_0^2n_{eff}^2-k_0^2n_c^2$, while n_s and n_c are the substrate and cover refractive index, respectively. Thus, from the multilayer structure generated from the trial parameter values and m-line measurement parameters, the spectra of effective indices can be calculated.

3. Applications

The method was demonstrated by applying it to reconstruct refractive index profile of various kinds of waveguides. A 5-layer step index waveguide with highest index layer embedded under the surface is used as the first numerical sample. This sample has original refractive index of n_s=1.5, n_c=1, n_1=n_3=1.51, n_2=1.52, and thickness of h_1=h_2=h_3=1.5µm, where layer 1 is just after the substrate. The effective indices of this sample at light wavelength of 0.85µm as calculated by using characteristic matrix mode solver[8] are 1.5141040940127035 and 1.5037200838307276 for TE modes; and 1.5140302800962591 and 1.5036138282123948 for TM modes. These effective indices values and the corresponding light wavelength and polarization are fed into the method as measured data and measurement parameters, respectively. The n_s and n_c are assumed to be known. The method was used to find optimal index and thickness of layer 1 until 3. The results obtained after 28 iterations are 1.5112703974976507, 1.5203435462326425, and 1.5099137245363798 for refractive index; and 1.44435888726013µm, 1.36590649507949µm, and 1.54893458478656µm for thickness of layer 1 until 3, respectively. The reconstructed index profile agrees to its original profile as shown in Fig. 3.

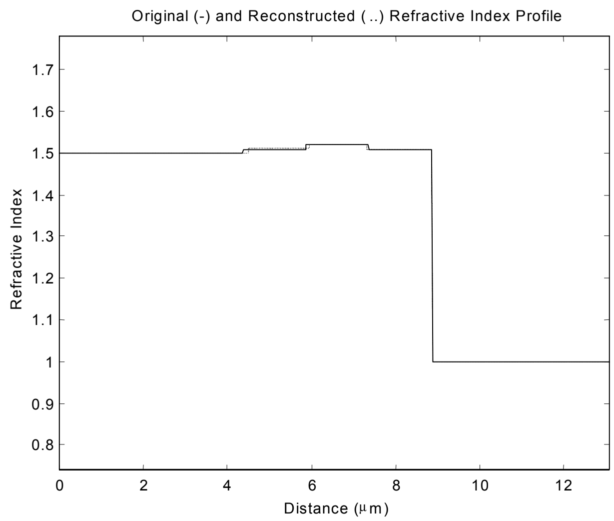

Figure 3. Original and reconstructed refractive index profile for dual-moded 5-layer step-index waveguide sample.

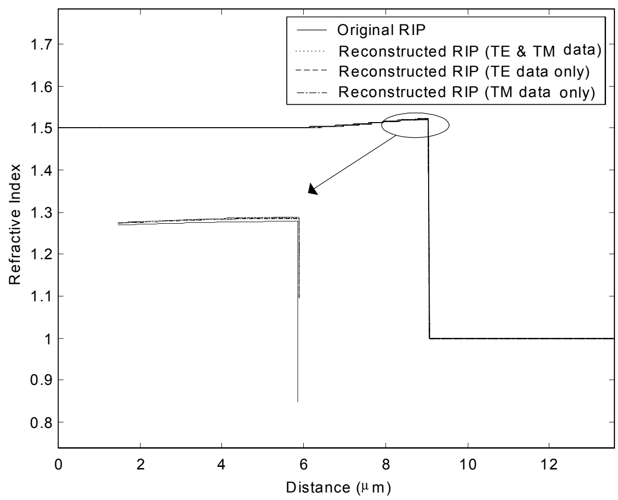

Figure 4. Original and reconstructed refractive index profile for sample with Gaussian index profile.

The method was also applied to sample with Gaussian index profile. This sample has original index profile function of $n(x)=n_s\{1+\Delta\exp[-(x/d)^2]\}$ below the surface with n_s=1.5, Δ=0.013333333, d=2µm, and air as the cover (n_c=1). At wavelength of 0.6328µm this sample support two TE modes with mode indices of 1.50982 and 1.5008 and two TM modes with mode indices of 1.50948 and 1.50062 as calculated by Chiang et.al. using WKB method[3]. The iterative Characteristic Matrix method was used to find optimal value of Δ and depth d of the sample by treating these mode indices values as measured data using both TE and TM mode indices, using TE mode indices only, and using TM mode indices only. In these reconstruction processes the Gaussian profile is approximated by 20-layer structure. The results fit to the original profile excellently, even when using data on one polarization orientation only as depicted in Fig. 4. The method is also used to reconstruct the refractive index profile of the same sample without using the analytic expression of its index profile. In this case the sample is treated as multilayer structure with 15 layers, and the method is used to find optimal refractive indices and thickness of these layers. The result is shown in Fig. 5. Although this under-determined case yields not so accurate results, but the reconstructed profile still resembles the original one.

The method is also succeeded to reconstruct refractive index profile of sample with exponential index profile. In this case we take the 2-moded sample with index profile function below the surface expressed as $n(x)=n_s[1+\Delta\exp(-x/d)]$ with original index profile parameters of n_s=2.177, Δ=0.019751952, d=0.931μm, and n_c=1. The mode indices for TE polarization as calculated using WKB method by Shiozawa et.al.[4] are 2.1910636 and 2.1793766. The reconstruction converged after 10 iterations yielding optimal index profile parameters of Δ=0.020009933639258532 and d=0.92442507001508μm. As shown in Fig. 6, the reconstructed profile almost coincides with the original one.

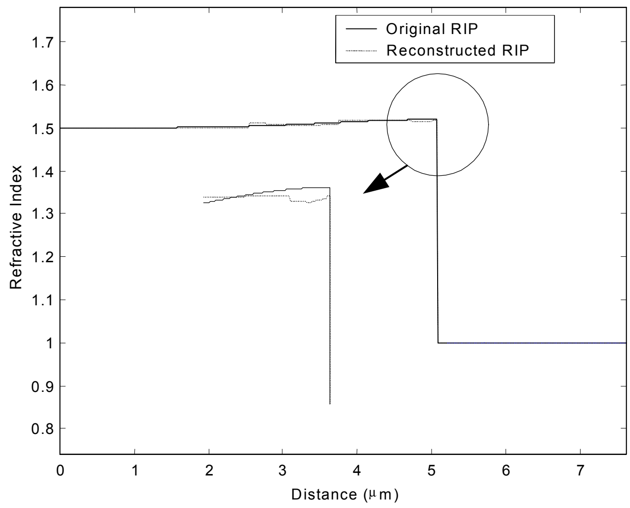

Figure 5. Reconstructed index profile of Gaussian sample without using analytic expression of its profile function.

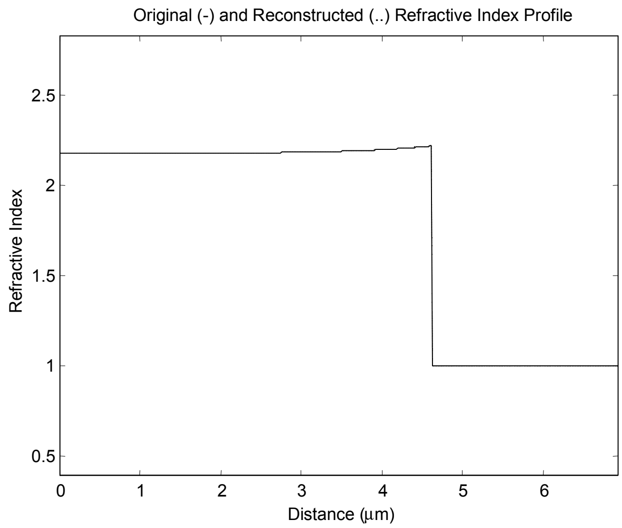

Figure 6. Reconstructed index profile for sample with exponential index profile.

4. Conclusions

A method to reconstruct the refractive index profile of planar waveguides from measured mode effective indices spectra using iterative Characteristic Matrix Method is described and demonstrated using samples of step and graded index profiles. The results agree with the original profiles. The possibility to reconstruct the index profile without knowing the profile function expression in advanced was also demonstrated.

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