Minimax Optimal Control and Filtering for Stochastic Uncertain Systems [*]

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Two important and related ideas to emerge in modern control theory are the Kalman Filter and the solution to the linear quadratic Gaussian (LQG) stochastic optimal control problem. However, the above techniques suffer from a major disadvantage in that they do not provide a systematic means for addressing the issue of robustness. Indeed, it is well known that an LQG controller may lead to very poor robustness with respect to the classical robustness measures of gain and phase margin. This fact has motivated a significant amount of interest in the problem of robust LQG control; e.g., see [1].

It is well known that the Kalman filter may lead to poor performance in the face of uncertainty in the underlying dynamic model; e.g., see [2]. The development of a robust version of the Kalman filter is an important and challenging problem. This problem is especially important in situations where one wishes to take into account the fact that the noise disturbances entering into the signal model differ from Gaussian white noise. For example, signals in telecommunication systems represent bitstreams of binary code and therefore have a distribution other than the Gaussian distribution. One possible approach addressing this issue is to consider the robust Kalman filtering problem as a problem of worst-case state estimation; e.g, see [3]. In this problem, one seeks to find an estimator minimizing the worst-case state estimation error in the face of uncertainty in the system.

A powerful idea which can be applied to many robust control and filtering problems emerges from the fact that such problems may be formulated as minimax optimization problems. This idea relies on the interpretation of the worst-case design problem in the presence of uncertainty as a game-type problem. In this game-type problem, the designer is considered as the “minimizing player” who endeavors to find an optimal control/filter strategy to maintain a certain guaranteed level of the performance of the closed loop system in the face of uncertainty. In contrast, the uncertainty in the underlying process impairs the performance of the closed loop system. This motivates us to think of the uncertainty as the “maximizing player” in a game problem. Normally, one must consider a specified class of uncertainties acting on the system. This class forms a pre-defined set of admissible uncertainty inputs from which the maximizing strategies are chosen. The advantage of this approach is that it may allow one to convert problems of robust control and filtering into mathematically tractable game-type minimax optimization problems; e.g., see [2, 1, 4]. In particular, for the robust control and filtering problems to be considered in this paper, this game-type optimization problem can be in turn converted into a tractable risk sensitive control or filtering problem; e.g., see [5, 6, 7, 8].

A suitable way of describing noise disturbances in problems of robust LQG control and robust Kalman filtering is to treat the disturbances as uncertain stochastic processes subject to constraints; e.g., see [4]. Formalizing this idea leads to an stochastic uncertain system framework which allows us to extend the standard LQG controller design methodology into a minimax optimal LQG controller design methodology; e.g., see [5, 6, 1]. Similarly this idea allows us to extend the standard Kalman filter design methodology into a minimax optimal stochastic filtering methodology for uncertain systems.

Our approach to minimax optimal stochastic filtering is to consider the robust filtering problem as a problem of worst-case estimation. In this problem, one seeks to find an estimator minimizing the worst-case error in the face of uncertainty which satisfies a certain stochastic uncertainty constraint. This con-

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straint describes restrictions on the *relative entropy* between an uncertain probability measure related to the distribution of the uncertain noise, and a reference probability measure corresponding to Gaussian white noise; e.g., see [5, 6, 7]. Such a constraint can be thought of as a stochastic counterpart of the deterministic integral quadratic constraint (IQC) uncertainty description. In particular, H^∞ norm-bounded linear time invariant uncertainty and cone-bounded nonlinear uncertainty can be incorporated into our framework by constructing a corresponding uncertain probability distribution.

The solution to the above minimax optimal state estimation problem is obtained by converting this problem into an equivalent partial information risk sensitive control problem. In particular, solutions to a parametrized risk-sensitive control problem provide us with an estimator which guarantees an upper bound on the time-averaged estimation error in the face of the admissible uncertainty. This leads to a robust Kalman filter synthesis procedure based on a pair of specially parametrized algebraic Riccati equations arising in deterministic H^∞ control and maximum entropy H^∞ control. The resulting robust estimator is therefore a state estimator of the type which occurs in H^∞ filtering.

As an illustration of the efficacy of our minimax optimal filtering approach, we apply the result discussed above to a problem arising in telecommunications systems. This problem is the problem of equalization (input estimation) for an intersymbol interference (ISI) channel. For this problem, the results of the paper give a guaranteed error input estimator which is robust against the uncertainty in the probability density of the input symbols.

As a conclusion, some potential applications to the area of hybrid systems are discussed. In particular, an important issue to be addressed concerns robust stochastic control and filtering for systems with Markovian jumping parameters (MJP). This class of systems arises in control systems facing randomly occurring hardware failures and other abrupt changes in the system structure. A large-scale computer network with randomly occuring information traffic jams may serve as another example of systems with *uncertain* MJP. One possible way of modeling such systems is to model the abrupt changes in the system structure using discrete state Markov processes. Systems with Markovian jumping parameters are sometimes viewed as a special type of hybrid systems. We focus on the possibility of an extension of our minimax filtering approach to systems with MJP.

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