Variable Period Deadbeat Control of High Order Chained Systems with Applications to the Control of Underactuated Manipulators

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In mechanical systems subject to nonholonomic constraints, we can control more number of generalized coordinates than the number of inputs by taking advantage of the constraints. If the constraints are expressed in terms of the velocity, control systems are usually described by symmetrical affine systems while they can be described by affine systems with drift terms when the constraints are given in terms of the acceleration.

Since almost all symmetrical affine systems of order third or fourth can be transformed into the so called chained system, many nonholonomic control laws are designed to control the chained systems. Furthermore, it is known that the settling problem to the reference state is converted to the stabilizing problem at the origin . However, due to the theorem of Brockett, there is no time-invariant and smooth feedback control law to stabilize the symmetrical affine systems. There are many research results to overcome this limitation including the multirate digital control proposed by Monaco.

Comparing to the symmetrical affine systems, there are few amount of studies on the control of the systems having drift terms. However, there are several studies on underactuated manipulators (UAM) which represent the mechanical systems subject to the acceleration constraint. Laiou et al. analyzed the so called high order chained systems with drift terms and derived control laws including one which is applicable to the control of a UAM.

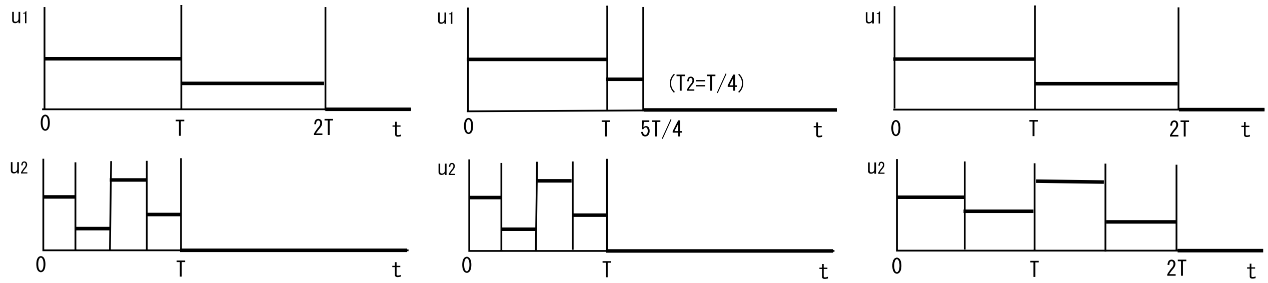
In this paper, we will focus on the multirate digital control. The multirate digital control can be easily seen to be applied to all the chained systems including the multi-chain systems and the high order multi-chain systems with drift terms as far as they have only single generators since all the sub-chains become linear when the generator variable is set to be non-zero constant. However, we has to use n_i sampling intervals for the n_i th order sub-chain system during one sampling interval of the generating system. This means that the sampling period of each sub-chain becomes shorter than the sampling period of the generating system. And this causes large overshoots for the input as well as some of the state variables of the sub-chain systems and they very often close or reach the singular manifold of the transformation introduced to transform given plants to the chained systems.

To overcome this imbalance of the sampling period, we will first develop here a variable speriod deadbeat control (VPDC) which is equivalent to a deadbeat control for time varying discrete-time systems. Using this method, we can make the sampling interval of the generating system as well as all sub-chain systems uniform to improve the transient responses and to get around the singularity problem. We will then apply VPDC to the control of a three link UAM which can be transformed into a high order chained system with the drift term.

Let's examine the following high order chained system

$$\begin{aligned} z_1^{(2)} &= u_1 \\ z_2^{(2)} &= u_2 \\ z_3^{(2)} &= z_2 u_1 \end{aligned} \tag{1}$$

If we will directly apply the multirate digital control, the two inputs u_1 and u_2 are generated as in Fig.1(a). First we can see that the second interval of u_1 is vain. Especially, we see that four sampling intervals are needed for u_2 during one sampling interval of u_1. In this sampling scheme, some of the variables of Σ_2 inevitably have unnecessarily large overshoots when T is chosen shorter.



(a) Direct extension of the multirate digital control **(b) Change of the interval for u_1** **(c) Change of the interval for u_2**

Fig.1 Improvement of the sampling interval

We cannot apply the conventional deadbeat control which makes the transition matrices nilpotent by assigning all poles at the origin because the sampling period of u_1 varies during control in the first case while the parameters of Σ_2 varies during control in the second case.

We can solve the problem by using the following theorem which gives a systematic design procedure of the deadbeat controller for time varying sampled data systems.

Theorem 1 *Suppose $x \in R^n$ and $u \in R^1$ and let*

$$\dot{x}(t) = \alpha_i x(t) + \beta_i u(t) : \quad T_{i-1} \leq t < T_i \tag{2}$$

be a continuous time piece-wise constant coefficient linear system and assume that α_i and β_i are constant and (α_i, β_i) is the controllable pair. Also let

$$\dot{x}_{i+1} = A_i x_i + \beta_i u_i \tag{3}$$

where

$$A_i = e^{\alpha_i T_i}, \quad b_i = \int_0^{T_i} e^{\alpha_i \tau} d\tau \beta_i \tag{4}$$

be the corresponding sampled data system discretized using the zero-order hold of sampling period T_i. Then for almost all sampling period T_i $(i = 1 \sim n)$, there exist feedback gain vectors $f_1 \sim f_n$ satisfying

$$(A_n - b_n f_n) \ldots . (A_2 - b_2 f_2)(A_1 - b_1 f_1) = 0 \tag{5}$$

As an application, we will deal with the control problem of a three link planar manipulator laid on the plane as shown in Fig.2 where the third (tip) link has no actuator to be controlled. Since the motion of the third link is convetrted into the chained from (1), we can apply Theorem 1 to improve the responses to get aroud the sigularity problem.

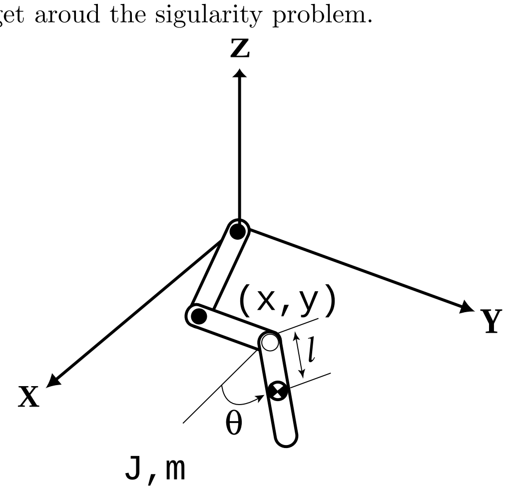


Fig. 2 UAM

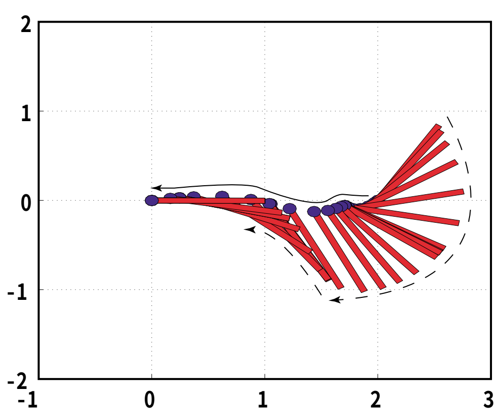


Fig. 3 Motion of the third link