Recent Results in performance limitations for Feedback Control.

R.H. Middleton*

September 25, 2000

1 Introduction and Motivation

There has been a long history, dating back to Bode [1], with more recent contributions such as [6], [13], [11], [14] and [4] which gives insights into limitations on the achievable performance of a feedback control system. We are particularly interested in limitations or constraints which are dictated by the plant itself, together with controller causality and closed loop stability. In some cases, the constraints are also based on linearity of the feedback controller itself. Ideally, the constraints should be expressible in terms which are fairly intuitive, lending insight into the structure of control problems.

At first sight, it may appear that this work on performance limitations is of little relevance to an ARO Workshop aimed at Intelligent, Adaptive and Hybrid systems. However, as argued below, the key contribution of results in fundamental limitations is to understand issues of architecture, hierarchy and structure in control systems which are at the heart of hybrid and hierarchical systems. To further motivate this relationship we first consider a number of philosophical claims about the nature of many control design problems, starting firstly adaptive and intelligent systems.

1.1 Essentially all feedback controllers are 'adaptive' and 'intelligent' to some extent

Many definitions of Adaptive or Intelligent control (see for example [18]) are based around ideas such as:

- feedback systems where the performance improves over time;
- feedback systems with the ability to deal with unforeseen changes in the system to be controlled, frequently by modelling aspects of the system, and compensating for them.

However, in both cases, very simple classical controllers such as PI controllers have these features to some extent, as illustrated in figure 1.

Whilst this may appear to be a trivial example of linear controllers being somewhat adaptive, a more subtle version of this is illustrated in figure 2

*Department of Electrical and Computer Engineering, The University of Newcastle, 2308 Australia, email: rick@ee.newcastle.edu.au

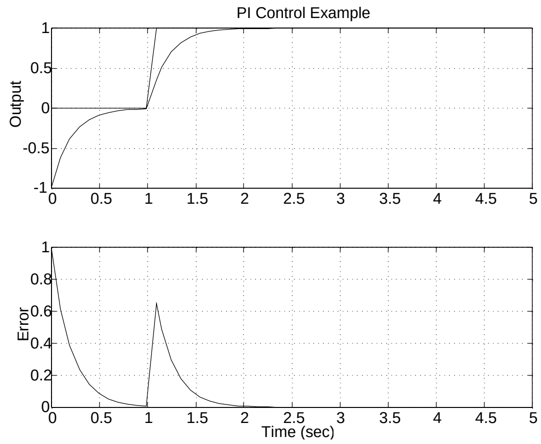

Figure 1: Example step responses for a closed loop system with PI control

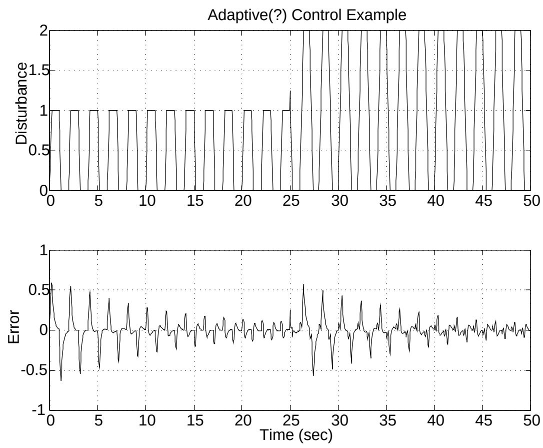

Figure 2: Example of an apparently adaptive system response.

Despite the fact that the performance shown in Figure 2 appears to improve with time, then deteriorates when conditions change, then improves again, the plant and controller for this example are in fact **linear time invariant** and would therefore not normally be classified as adaptive. Furthermore, for the particular controller, qualitatively similar behaviour can be seen for some plant parameter changes, that is, the system appears to adapt to plant and disturbance changes.

This suggests the following key questions for adaptive systems:

1. When is the 'adaptation' and 'intelligence' of a well designed linear controller adequate?
2. If a controller is to be made adaptive, over what time scale, and to what types of system changes is it required to adapt to?

The theory of fundamental limitations on control performance is useful for helping to answer the first kind of question, including in some cases being able to make statements about the limits of achievable performance with a nonlinear controller. One way to assess the achievable performance of a feedback control system is to perform an optimisation based controller design, and examine the resulting achieved cost. Whilst optimisation is a very powerful tool for control design, we believe it is not a panacea for design. In particular, we claim:

1.2 The majority of control solutions are not achieved via a single process of performance and constraint specification, followed by optimisation

This is a claim which is perhaps difficult to either substantiate or falsify. However, to add at least some support to this statement, observe that many control designs, even those employing the powerful tools of optimal control, often proceed by iterations of careful choice of user parameters and simulation or experimentation with the resulting controller[1]. One conclusion which may be drawn from this is that it is rarely possible to a priori specify an appropriate cost function for a control problem, rather, considerable insight and some iteration is frequently required to generate an appropriate cost function. In addition, this suggests that many real control problems have multiple objectives and involve inevitable trade-offs between competing objectives. In this context, the role of the theory of fundamental limitations is to guide the control designer into the key factors which limit performance. This can help avoid excessive iterations in the design phase where continued attempts to improve performance by 'tweaking' weightings may be fruitless.

The following claim further undermines a naive view of optimisation theory as solving all control design problems.

1.3 At least from a theoretical viewpoint, some simple robust control design problems are known to be computationally intractable

It is known that in general, the problem of designing a controller to achieve robust performance with parametric uncertainty is NP-hard [2]. Furthermore, several closely related problems can also be shown to be NP-hard [8]. This strongly suggests that in general, high performance control design for complex systems

[1]These statements do not imply that optimal control tools are not useful. On the contrary, without such tools, many control problems would become completely intractable. Arguably, one of the great strengths of optimal controls is that they offer the control designer 'tuning' knobs in the optimisation weights that have simple and intuitive relationships to important aspects of the control system performance.

must involve suitable suboptimal heuristics, decomposition into simpler problems, and hierarchical design. By illuminating in a fairly simple and intuitive fashion structural features of a control design problem, the theory of fundamental limitations helps inform the design process, and helps the designer select suitable control architectures.

Despite this, however, we note in some cases controllers, though perhaps fairly complex, with almost 'optimal' performance, can be easily designed using classical techniques.

1.4 Some control problems can be solved with controllers of a simple structure whilst achieving almost optimal performance

In some cases, the feedback control performance is limited by fundamental properties of the plant itself. In such cases, the theory of fundamental performance limitations may quickly indicate whether a simple controller is achieving performance close to that achievable with any controller. In such cases, much wasted effort in control loop design can be avoided, and attention focused on other issues.

For example, in the IFAC 1999 World Congress in Beijing, there was a benchmark industrial control problem posed for an atomic reactor drum level control problem. A key part of the problem was a disturbance rejection specification, for a range of operating points of the reactor. It turned out that the system, at most operating points, had a significant non-minimum phase zero, which limits the achievable step disturbance performance[2]. Having computed a lower bound on the achievable performance, we could easily show that we were able, with classical control techniques, achieve performance close to optimal, and therefore further control design work and complexity was not warranted. Figure 3 illustrates this where the performance for our example controller (gain scheduled, cascaded PIs loops, with disturbance feedforward) (shown by the + marks) is compared against the theoretically best achievable performance [17].

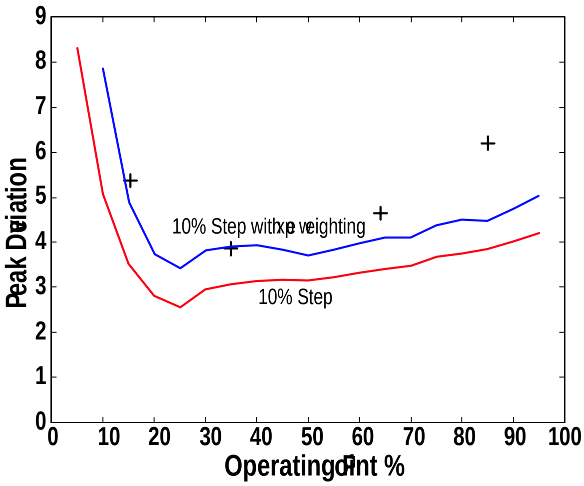

Figure 3: Comparison of peak deviation in response to a step disturbance; actual control performance '+'; performance limit: solid curves.

[2]In this particular case, the fundamental limits on performance apply to all types of controllers, including nonlinear, adaptive, intelligent and so on.

2 Recent Results in Performance Limitations

Here we wish to highlight some of the results which fit the category of fundamental performance limitations. These results are somewhat diverse, and are given below without any particular order.

2.1 Non Right Invertible Systems

Some recent results have examined the situation where we have a 'tall' plant, in other words, a plant with more performance variables than actuators, also called 'non right invertible'. This limitation can be examined in the form of frequency domain integral constraints [15], similar in nature to the Bode sensitivity integral, or also as lower bounds on the achievable quadratic error performance. In both cases, the key results give that a dynamic, or non-static relationship between performance variables for a control problem, places a lower limit on the achievable control performance. This can from cheap control results such as:

Claim 1 [16] Consider the problem of tracking a unit step reference for a SITO plant with transfer function $P(s) = \begin{bmatrix} p_1(s) & p_2(s) \end{bmatrix}^T$. Without loss of generality, perform an output transformation so that $P(0) = \begin{bmatrix} p_1(0) & 0 \end{bmatrix}^T$. Then for any stable state feedback system for a SITO plant tracking a unit step set point in the first output:

$$\int_0^\infty e^T(t)\, e(t)\, dt \geq Var(P(s)) + 2 \sum_{z_i \in NMP} \frac{1}{z_i} \tag{1}$$

where $Var(P(s))$ is a non-negative term capturing the variation of the direction of the transfer function with frequency, which is zero if and only if the range space of $P(j\omega)$ is independent of ω; and z_i are the non minimum phase zeros of $p_1(s)$.

Furthermore, it can be shown (e.g. [3]) that qualitatively similar results hold for the nonlinear case, where having more performance variables than control variables places a non-trivial lower limit on the achievable tracking performance, even when the plant is otherwise minimum phase.

2.2 Ill Conditioned Plants

Ill conditioned plants are known to pose robustness and performance control difficulties. It is known that Ill conditioning can arise due to poor scaling in a plant (e.g. by inappropriate choice of units), or due to (for example) the columns of the plant having almost coincident range spaces. Recent results have better delineated this difficulty. For simplicity, consider a two input two output plant, with transfer function at a particular frequency given by $P(j\omega) = \begin{bmatrix} P_{\cdot 1} & P_{\cdot 2} \end{bmatrix}$. Without loss of generality, take $||P_{\cdot 1}|| \geq ||P_{\cdot 2}||$. We then have the following:

Claim 2 [9] The condition number of the plant transfer function, $\kappa(P(j\omega))$ satisfies:

$$\left(\frac{||P_{\cdot 1}||}{||P_{\cdot 2}||} + \frac{||P_{\cdot 2}||}{||P_{\cdot 1}||} \right) \frac{1}{\sin(\phi)} \geq \kappa \geq \frac{||P_{\cdot 1}||}{||P_{\cdot 2}||} \frac{1}{\sin(\phi)} \tag{2}$$

where ϕ is the angle between the vectors $P_{\cdot 1}$ and $P_{\cdot 2}$.

Other results consider the implications of poor conditioning on stability robustness, decoupling control and the relationship to the relative gain array. These results highlight some of the fundamental difficulties in controlling an ill conditioned plant. In particular, it highlights difficulties inherent in the case where we have 'almost redundant actuators', namely, where $\phi \approx 0$.

2.3 Preview Control for NMP plants

Preview Control (see for example [10]) denotes the situation where a reference command or disturbance is available as a measured signal, with a finite time advance. In such cases, it can be shown that many of the tracking difficulties for non-minimum phase systems can be ameliorated. Consider, for example, the problem of tracking a reference signal $r(t)$ where a preview of the signal of T seconds is available. Suppose further that the controller is permitted to be a linear time invariant two degree of freedom controller, and denote by $S(s)$ the transfer function from the reference signal to the error $e(t)$ between the reference and the plant output. Suppose that the SISO plant has a single real NMP zero at $s = z$.

Claim 3 (Detailed Proof is in preparation) For the situation described above, for any internally stabilising controller $S(s)$ must satisfy:

$$\int_0^\infty \log |S(j\omega)| \frac{2z}{z^2 + \omega^2} d\omega \geq -zT \tag{3}$$

Furthermore, the inequality, (3) is tight in the sense that for any $\varepsilon > 0$ there exists a controller which achieves:

$$\int_0^\infty \log |S(j\omega)| \frac{2z}{z^2 + \omega^2} d\omega \leq \varepsilon - zT \tag{4}$$

Note that in the absence of preview, the above inequality basically[3] restates the well known Poisson integral [6]. This integral imposes an upper bound on the bandwidth over which sensitivity reduction can be achieved, without causing excessive high frequency sensitivity peaks. However, when we have preview (that is, $T > 0$), Claim 3 shows that this constraint is ameliorated. In particular, if $T >> \frac{1}{z}$ then the integral inequality constraint becomes very weak.

2.4 Feedback Data Rate Required for Stability

Consider a SISO discrete time linear plant, where there is a communication constraint on the information transfer. Denote the discrete time plant transfer function by $P(z)$. We then have:

Claim 4 [12] Let $|p|_{\max}$ denote the maximum magnitude of an unstable pole of $P(z)$. Then $P(z)$ is stabilisable by a controller with finite data communication rate if and only if the bit rate, R (in bits per sample) satisfies:

$$R > \log_2 |p|_{\max} \tag{5}$$

[3]Note that the terms due to open loop unstable plant poles included in the regular Poisson integral are not required in this case since we allow two degree of freedom control.

2.5 Ability of NL/TV to do Better in Some Cases

It has been known for some time that special forms of integrators (for example) allow better overall control performance in some cases. The special forms typically take the form of resetting or saturation under particular conditions, and thereby turn the feedback control system into a hybrid, or switched system. By incorporating in such schemes prior knowledge of the class of systems to be controlled, improved performance is obtained. For example, in [5], knowledge of the maximum size of an input disturbance allows use of a resetting integrator to obtain improved performance, even though the plant itself is linear time invariant.

3 Conclusions

In this paper, we have briefly reviewed two seemingly disparate areas of logarithmic sensitivity integrals and limiting linear quadratic optimal control problems. The results of this paper link these two areas, and in particular provide a direct link between minimum energy LQ control, and the Bode Sensitivity Integral. Dual results establish a direct link between cheap control problems, and the complementary sensitivity integral.

References

[1] H. Bode, "Network Analysis and Synthesis", New York, Von Nostrand 1945.

[2] R.P. Braatz et al, "Computational Complexity of μCalculation", IEEE transaction on Automatic Control, V39, N5, pp1000-1002, May 1994.

[3] J.H. Braslavsky, R.H. Middleton and J.S. Freudenberg, "Performance Limitations in a Class of Single Input Two Output Nonlinear Systems", 1999 American Control Conference.

[4] Chen, J., O. Toker and L. Qiu, "Limitations on Maximal tracking Accuracy", IEEE transactions on Automatic Control, V45, N2, pp326-331, 2000.

[5] A. Feuer, G.C. Goodwin and M. Salgado, "Potential Benefits of Hybrid Control for Linear Time Invariant Plants", Proc. American Control Conference, 1999.

[6] J.S. Freudenberg and D. Looze, "Right half plane poles and zeros and design trade-offs in Feedback systems", IEEE Transactions on Automatic Control, AC-30, pp555-565, 1985.

[7] J.S. Freudenberg and D. Looze, "Frequency Domain Properties of Scalar and Multivariable Feedback Systems", Vol. 104, Lecture Notes in Control and Information Sciences, Springer-Verlag, 1988.

[8] M. Fu, "The Real Structured Singular Value is Hardly Approximable," IEEE Trans. Auto. Control, vol. 42, no. 9, pp. 1286-1288, 1997.

[9] J.S. Freudenberg and R.H. Middleton, "Feedback Systems with an almost rank deficient Plant", 1999 American Control Conference.

[10] Kojima, A. and S. Ishijima, "H_∞preview disturbance attenuation", Proc. 37th IEEE Conference on Decision and Control, Tampa, Florida, 1998.

[11] R.H. Middleton, "Trade offs in linear Control System Design", Automatica, V27, pp281-292, 1991.

[12] G.N. Nair and R.J. Evans, "Optimal Communication-Limited Control and Estimation", Technical Report, Department of Electrical and Electronic Engineering, The University of Melbourne, VIC 3010 Australia.

[13] L. Qiu and E.J. Davison. "Performance limitations of non-minimum phase systems in the servomechanism problem." Automatica, 29(2):337–349, 1993.

[14] M.M. Seron, J.H. Braslavsky and G.C. Goodwin, "Fundamental Limitations in Filtering and Control" Springer-Verlag, 1997.

[15] A.R. Woodyatt, J.S. Freudenberg and R.H. Middleton, "An Integral constraint for Single Input Two Output Systems", to appear, Automatica.

[16] A.R. Woodyatt, M.M. Seron, J.S. Freudenberg and R.H. Middleton, "Cheap Control Tracking Performance for Non-Right Invertible Systems", submitted for publication, IEEE Transactions on Automatic Control

[17] J. Ward and R.H. Middleton, "Sequential approach to control systems synthesis with constraints", 1999 IFAC World Congress, Beijng

[18] G. Zames, "Adaptation and Identification: Towards a General, Complexity-Based Theory", Plenary Lecture, 1996 Conference on Decision and Control