Geometric Results on the Well-Posedness of Piecewise Linear Systems

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Abstract

This paper supplements the recent results of Imura and van der Shaft on the well-posedness of a class of piecewise linear systems. We give a set of necessary and sufficient geometric conditions for well-posedness of a bimodal piecewise linear system with an affine switching surface.

1 Introduction

In a recent paper [1] Imura and van der Shaft have given a nice analysis of the well-posedness (existence and uniqueness of solutions) of a class of piecewise linear (discontinuous) systems. This paper addresses this issue for bimodal piecewise linear systems and supplements the results of [1] with a set of necessary and sufficient geometric conditions for well-posedness.

2 Bimodal Systems

Following [1], a piecewise linear bimodal system is defined by

$$\Sigma_O\colon \quad \dot{x} = \begin{cases} Ax, & \text{if } c^T x \geq 0 \text{ (mode 1)} \\ Bx, & \text{if } c^T x \leq 0 \text{ (mode 2)} \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$, $A, B \in \mathbf{R}^{n \times n}$ and $c \in \mathbf{R}^n$. More formally the system Σ_O is given in integral form by

$$x(t) = x_0 + \int_{t_0}^{t} f(x(s))ds \tag{2}$$

where $f(x)$ is the discontinuous vector field given by the right hand side of (1) and $x(t_o) = x_o$. If $x(t)$ satisfies (2) and t is a time when the the vector field switches from mode 1 to mode 2, then t is called an *event time* of the trajectory. A point t^* is a *right (resp. left) accumulation point* of the trajectory, if there exists a sequence of event times t_j with $t_j < t_j + 1$ (resp. $t_j > t_{j+1}$) such that $t_j \uparrow t^*$ (resp. $t_j \downarrow t^*$). A solution of Σ_O on the time interval $[t_o, t_1)$ from initial state x_o at time t_o is an absolutely continuous function satisfying (2) with no left accumulation points in $[t_o, t_1)$. The system Σ_O is *well-posed* if there exists a unique solution on $[0, \infty)$ for every initial state [1].

In [1] well-posedness is characterized in terms of the following concept. *Smooth continuation* with respect to $S \subset \mathbf{R}^n$ is possible from $x_o \in S$ if there exists $\epsilon > 0$ such that $x(t)$ is a solution of Σ_O over $[0, \epsilon)$ and $x(t) \in S$ for all $t \in [0, \epsilon)$. The system Σ_O has a smooth continuation with respect to S it it has a smooth continuation with respect to each $x_o \in S$.

Lemma 2.1 *The following statements are equivalent:*

(1) The system Σ_O is well-posed;

(2) From every initial state $x_o \in \mathbf{R}^n$ either:

- *a) smooth continuation is possible with respect to exactly one of the the modes, i.e., with respect to $\{x : c^T x \geq 0\}$ or $\{x : c^T x \leq 0\}$; or*
- *b) smooth continuation is possible over $[0, \epsilon)$ with respect to both modes and both solutions are identical over this time interval.*

Proof: See [1]. $\diamondsuit$

From this point on we will abbreviate 'smooth continuation' to simply 'continuation'.

3 Geometric Conditions for Well-Posedness

First some notation. For $M = A, B$ let $\mathcal{N}^j_{c,M} = \cap^j_{i=0}\ker(c^T M^j)$, $j = 0, \ldots, n-1$, and let $\mathcal{N}_{c,M} = \cap^{n-1}_{j=0}\mathcal{N}^j_M$. Hence $\ker(c^T) = \mathcal{N}^0_{c,M} \supseteq \mathcal{N}^1_{c,M} \supseteq \cdots \supseteq \mathcal{N}^{n-1}_{c,M} = \mathcal{N}_{c,M}$. Let $k_M = \min\{k \geq 0 : \mathcal{N}^k_{c,A} = \mathcal{N}_{c,A}\}$ and $\mathcal{N}_0 \triangleq \mathcal{N}^0_{c,A} = \mathcal{N}^0_{c,B} = \ker(c^T)$. $\mathcal{N}^0$ is called the *switching manifold*. The subspaces $\mathcal{N}^j_{c,A}$ and $\mathcal{N}^j_{c,B}$, $j = 1, \ldots, n-1$, are all subspaces of the switching manifold. In particular, $\mathcal{N}_{c,A} = \mathcal{N}^{k_A}_{c,A}$ and $\mathcal{N}_{c,B} = \mathcal{N}^{k_B}_{c,B}$ are the unobservable subspaces of the pairs (c^T, A) and (c^T, B), respectively. Finally, for $x_o \in \mathbf{R}^n$ let $x_M(t, x_o)$ denote the solution over $[0, \infty)$ of the linear system $\dot{x}(t) = Mx(t)$ with $x(0) = x_o$.

We first determine a set of necessary geometric conditions for well-posedness of Σ_O. Let $x_o \in \mathcal{N}^0$. The simplest case is when neither Ax_o nor Bx_o are in the null space of c^T. In this case, both Ax_o and Bx_o must point into the same mode space. Hence the inner product of the normal to the switching manifold at x_o, i.e., c, and the two vector fields at x_o both have the same sign: $\operatorname{sgn}(c^T A x_o) = \operatorname{sgn}(c^T B x_o)$. If these terms have different signs, then either there are two distinct valid trajectories or there is no valid trajectory from x_o. To complete this geometric picture we need to consider initial states $x_o \in \mathcal{N}^0$ with $x_A(t, x_o)$ or $x_B(t, x_o)$ tangent to the switching manifold. We do so in the following sequence of lemmas.

Lemma 3.1 *If Σ_O is well-posed, then $\mathcal{N}_{c,A} = \mathcal{N}_{c,B}$.*

Proof: Let $x_o \in \mathcal{N}_{c,A}$ and $x_o \notin \mathcal{N}_{c,B}$. Then $x_o \neq 0$ and $x_A(t, x_o) \in \mathcal{N}^0$ for all $t \geq 0$. The trajectory $x_B(t, x_o)$ must lie on one side of the switching surface for some interval $t \in (0, \epsilon)$. Since there is a valid local continuation from x_o with respect to mode 1, $c^T x_B(t, x_o) > 0$ for $t \in (0, \epsilon)$. But then from $-x_o$ we have two valid continuations: $x_A(t, -x_o)$ satisfies $c^T x_A(t, -x_o) = 0$ and for some $\epsilon > 0$, $x_B(t, -x_o)$ satisfies $c^T x_B(t, -x_o) < 0$ for $t \in (0, \epsilon)$. A contradiction. Thus $\mathcal{N}_{c,A} \subseteq \mathcal{N}_{c,B}$. A symmetric argument shows that $\mathcal{N}_{c,B} \subseteq \mathcal{N}_{c,A}$. $\diamondsuit$

When Σ_O is well-posed, let $\mathcal{N} \triangleq \mathcal{N}_{c,A} = \mathcal{N}_{c,B}$. For $M = A, B$ let $M|\mathcal{N}$ denote the linear map M restricted to the linear subspace $\mathcal{N}$.

Lemma 3.2 *If Σ_O is well-posed, then $A|\mathcal{N} = B|\mathcal{N}$.*

Proof: Let $x_o \in \mathcal{N}$. Clearly, for $M = A, B$, $x_M(t, x_o) \in \mathcal{N}^0$ for all $t \geq 0$. Hence by well-posedness both trajectories must be equal. Since this holds for every state in $\mathcal{N}$, the result follows. $\diamondsuit$

Next we consider trajectories that leave the switching manifold. The direction in which they do so is determined by the first nonzero derivative of the trajectory, i.e., by the least k, with $0 \leq k \leq n-1$, for which $x_o \in \mathcal{N}^{k-1}_{c,M}$ but $x_o \notin \mathcal{N}^k_{c,M}$, $M = A, B$. Well-posedness forces these subspaces to be the same for each mode. This the content of the next lemma.

Lemma 3.3 *If Σ_O is well-posed then, $k_A = k_B$ and for each j, $0 \leq j \leq k_A$, $\mathcal{N}^j_{c,A} = \mathcal{N}^j_{c,B}$.*

Proof: We use induction on j. The second result holds for $j=0$. Assume $\mathcal{N}_{c,A}^{j-1}=\mathcal{N}_{c,B}^{j-1}$. Suppose that $x_o \in \mathcal{N}_{c,A}^{j}$ and $x_o \notin \mathcal{N}_{c,B}^{j}$. Then $x_o \in \mathcal{N}_{c,A}^{j-1}=\mathcal{N}_{c,B}^{j-1}$. Consider a small open ball in $\mathcal{N}_{c,A}^{j-1}$ about x_o. This ball contains points z satisfying $c^T A^j z>0$ as well as points z satisfying $c^T A^j z<0$. If $c^T A^j z>0$ (resp. $c^T A^j z<0$) , then for some $\epsilon>0$ the trajectory $x_A(t,z)$ satisfies $c^T x(t,z)>0$ (resp. $c^T x(t,z)<0$) for all $t \in (0,\epsilon)$. Since $c^T B^j x_o \neq 0$ and the set where this holds within $\mathcal{N}_{c,A}^{j-1}$ is open, by making the ball around x_o sufficiently small we can ensure that for all z in the ball, there exists an $\epsilon>0$ such that $c^T x_B(t,z)$ has the same sign for $t \in (0,\epsilon)$. But this implies that there is some z in the ball for which the trajectories $x_A(t,z)$ and $x_B(t,z)$, $t \in (0,\epsilon)$, lie in different modes; A contradiction. Thus $\mathcal{N}_{c,A}^{j}=\mathcal{N}_{c,B}^{j}$. It follows that $k_A=k_B$. $\diamondsuit$

Now that we have the above structural constraints we can return to the sign conditions on the derivatives of the trajectories. When Σ_O is well-posed, let $\bar{k} \triangleq k_A=k_B$ and $\mathcal{N}^j \triangleq \mathcal{N}_{c,A}^{j}=\mathcal{N}_{c,B}^{j}$, $j=0,\ldots,\bar{k}$.

Lemma 3.4 *If Σ_O is well-posed, then for $1 \leq j \leq \bar{k}$ and for each $x_o \in \mathcal{N}^{j-1}$ with $x_o \notin \mathcal{N}^j$,* $\operatorname{sgn}(c^T A^j x_o)=\operatorname{sgn}(c^T B^j x_o)$.

Proof: For $x_o \in \mathcal{N}^{j-1} \cap (\mathcal{N}^j)^c$, $c^T x_A(t,x_o)=c^T \sum_{i=0}^{\infty} A^i t^i x_o/i! = c^T A^j x_o t^j/j! + \text{H.O.T.s}$. Thus for sufficiently small $\epsilon>0$, the sign of $c^T x_A(t,x_o)$, $t \in (0,\epsilon)$, is determined by the sign of $c^T A^j x_o$. Similarly, the sign of $c^T x_B(t,x_o)$, for $t \in (0,\epsilon)$, is determined by the sign of $c^T B^j x_o$. Well-posedness requires these signs be the same. $\diamondsuit$

3.1 Necessary and Sufficient Conditions

Now we put the results of the previous lemmas together to obtain a set of necessary and sufficient geometric conditions for well-posedness.

Theorem 3.5 *The bimodal system Σ_O is well-posed if and only if each of the following conditions is satisfied:*

1) $k_A=k_B \triangleq \bar{k}$;

2) For each $0 \leq j \leq \bar{k}$, $\mathcal{N}_{c,A}^{j}=\mathcal{N}_{c,B}^{j} \triangleq \mathcal{N}^j$;

3) $A|\mathcal{N}^{\bar{k}}=B|\mathcal{N}^{\bar{k}}$;

4) For each $x_o \in \mathcal{N}^{j-1}$ with $x_o \notin \mathcal{N}^j$, $\operatorname{sgn}(c^T A^j x_o)=\operatorname{sgn}(c^T B^j x_o)$, $0 \leq j \leq \bar{k}$.

Proof: Necessity has been demonstrated in the previous lemmas. Sufficiency follows by demonstrating a continuation at $x_o \in \mathcal{N}^0$. First consider $x_o \in \mathcal{N}^{j-1} \cap (\mathcal{N}^j)^c$, $1 \leq j \leq \bar{k}$. Since $x_o \in \mathcal{N}^{j-1}$ and $x_o \notin \mathcal{N}^j$, the sign of $c^T x_A(t,x_o)$ for small t is determined by the sign of $c^T A^j x_o$. Similarly, the sign of $c^T x_B(t,x_o)$ for small t is determined by the sign of $c^T B^j x_o$. For well-posedness at x_o it is sufficient that these signs be the same. This is ensured by 4). Finally, for $x_o \in \mathcal{N}$, 3) ensures that the trajectories under each mode are identical. $\diamondsuit$

The verification of conditions 1)-3) of Theorem 3.5 requires standard algebraic tests. Condition 4) can be verified by testing the sign condition at $\bar{k}$ states. This is the result of the following corollary.

Corollary 3.6 *The bimodal system Σ_O is well-posed if and only if conditions 1)-3) of Theorem 3.5 hold and for any set of $\bar{k}$ points x_j with $x_j \in N^{j-1} \cap (\mathcal{N}^j)^c$, $j=1,\ldots,\bar{k}$,*

$$\operatorname{sgn}\left(\, c^T A x_1 \quad c^T A^2 x_2 \quad \cdots \quad c^T A^{\bar{k}} x_{\bar{k}} \,\right) = \operatorname{sgn}\left(\, c^T B x_1 \quad c^T B^2 x_2 \quad \cdots \quad c^T B^{\bar{k}} x_{\bar{k}} \,\right) \tag{3}$$

Proof: (Outline) Necessity is clear. Sufficiency follows if (3) implies condition 4) of Theorem 3.5. To show this use linearity of the vector fields and the fact that the sign test is a linear functional. $\diamondsuit$

For $n>2$ the well-posedness of the bimodal system Σ_O is not robust with respect to small *arbitrary* perturbations in A and B. The equality of the subspaces $\mathcal{N}_{c,A}^{1}=\mathcal{N}_{c,B}^{1}$ requires $\ker(c^T) \cap \ker(c^T A) = \ker(c^T) \cap \ker(c^T B)$. For $n>2$ these subspaces are generically not the zero subspace. In this case, small perturbations of B can change the orientation of $\ker(c^T B)$, and hence of $\ker(c^T) \cap \ker(c^T B)$, thus violating the conditions required for well-posedness.

3.2 Examples

The following examples are taken from [1]. The first models a collision with an elastic wall. Let

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ -k & -d \end{pmatrix} \quad c^T = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

Then $\ker(c^T) = \text{span}\{(0,1)^T\}$, and $\mathcal{N}^1_{c,A} = \mathcal{N}^1_{c,B} = \{0\}$. Thus $k_A = k_B = 1$ and conditions 1)-3) of Theorem 3.5 are satisfied. To check condition 4) select $x_1 = (0,1)^T$. Then $\text{sgn}(c^T A x_1) = 1 = \text{sgn}(c^T B x_1)$. Thus the system is well-posed.

The second example is a variation on the first. Let

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \quad c^T = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

Then $\ker(c^T) = \text{span}\{(0,1)^T\}$, and $\mathcal{N}^1_{c,A} = \mathcal{N}^1_{c,B} = \{0\}$. Thus $k_A = k_B = 1$ and conditions 1)-3) of Theorem 3.5 are satisfied. To check condition 4) select $x_1 = (0,1)^T$. This time $\text{sgn}(c^T A x_1) = 1 \neq -1 = \text{sgn}(c^T B x_1)$. Thus the system is not well-posed.

The third example models an elastic collision between two objects (Example 4.1 of [1]).

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -k_2 & -d_2 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -k_1 & -d_1 & k_1 & d_1 \\ 0 & 0 & 0 & 1 \\ k_1 & d_1 & -k_1 - k_2 & -d_1 - d_2 \end{pmatrix} \quad c^T = \begin{pmatrix} 1 & 0 - 1 & 0 \end{pmatrix}$$

For $k_2 \neq 0$ elementary calculation yields:

$$\mathcal{N}^1_{c,A} = \mathcal{N}^1_{c,B} = \text{span}\{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}\} \quad \mathcal{N}^2_{c,A} = \mathcal{N}^2_{c,B} = \text{span}\{\begin{pmatrix} -d_2/k_2 \\ 1 \\ -d_2/k_2 \\ 1 \end{pmatrix}\} \quad \mathcal{N}^3_{c,A} = \mathcal{N}^3_{c,B} = 0$$

So $\mathcal{N}_{c,A} = \mathcal{N}_{c,B} = 0$ and $A|\mathcal{N}_{c,A} = B|\mathcal{N}_{c,B}$. Thus the structural conditions of Theorem 3.5 are met. To check the sign conditions we use Corollary 3.6 with $x_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}^T$, $x_2 = \begin{pmatrix} \alpha & 1 & \alpha & 1 \end{pmatrix}^T$, $x_2 = \begin{pmatrix} \beta & 1 & \beta & 1 \end{pmatrix}^T$ where $\alpha \neq -d_2/k_2$ and $\beta = -d_2/k_2$. This yields $\text{sgn}\begin{pmatrix} c^T A x_1 & c^T A^2 x_2 & c^T A^3 x_3 \end{pmatrix} = \text{sgn}\begin{pmatrix} 1 & \alpha k_2 + d_2 & k_2 \end{pmatrix} = \text{sgn}\begin{pmatrix} c^T B x_1 & c^T B^2 x_2 & c^T B^3 x_3 \end{pmatrix}$. Thus the system is well-posed. The same conclusion holds when $k_2 = 0$. The computations are similar and hence omitted.

4 Affine Switching Manifold

Now consider an affine switching manifold. In this case the system becomes:

$$\Omega_O\colon \quad \dot{x} = \begin{cases} Ax, & \text{if } c^T x \geq d \text{ (mode 1)} \\ Bx, & \text{if } c^T x \leq d \text{ (mode 2)} \end{cases} \tag{4}$$

where $x \in \mathbf{R}^n$, $A, B \in \mathbf{R}^{n \times n}$, $c \in \mathbf{R}^n$ and $d \in R$. Let $S = \{x\colon c^T x = d\}$ and $h = dc/(c^T c)$. S is the switching manifold and h is the normal from the origin to S. Any $x_o \in S$ can be uniquely written as $x_o = h + z_o$ where $z_o \in \ker(c^T)$. As before, the well-posedness of Ω_O is determined by the existence of unique, mode-consistent continuations from the initial states on S. We will determine a set of necessary conditions for the well-posedness of Ω_O. We begin with the following preliminary result.

Lemma 4.1 *For $M = A, B$, if $Mc \notin \mathcal{N}_{c,M}$, there exists a least positive integer $1 \leq j_M \leq n$ such that $c^T M c \neq 0$. Conversely, if $Mc \in \mathcal{N}_{c,M}$, then M is singular and for all $i \geq 1$, $c^T M^i c = 0$.*

Proof: Omitted. ♢

For $x_o \in S$ write $x_o = h + z_o$ for $z_o \in \mathcal{N}^0$. Then for $M = A, B$, $x_M(t, x_o) = x_M(t, h) + x_M(t, z_o)$. If $Mc \in \mathcal{N}_{c,M}$, then for all t, $c^T x_M(t,h) = d$ and $\text{sgn}(c^T x_M(t, x_o) - d)$ is determined by $\text{sgn}(c^T x_M(t, z_o))$. In particular, if $Ac \in \mathcal{N}_{c,A}$ and $Bc \in \mathcal{N}_{c,B}$, the well-posedness of Ω_O is completely determined by the well-posedness of Σ_O. The more interesting situation is when $Mc \notin \mathcal{N}_{c,M}$ for at least one of $M = A, B$.

For simplicity of the presentation we will assume henceforth that both (c^T, A) and (c^T, B) are observable and that for at least one of $M = A, B$, $Mc \notin \mathcal{N}_{c,M}$. Let $\bar{j} = \min\{j_A, j_B\}$.

Lemma 4.2 *If Ω_O is well-posed then, for each $1 \leq j \leq \bar{j}$, $\mathcal{N}^j_{c,A} = \mathcal{N}^j_{c,B}$ and for each $z_o \in \mathcal{N}^j_{c,A}$, $\text{sgn}(c^T A^j z_o) = \text{sgn}(c^T B^j z_o)$.*

Proof: (Outline) Note $c^T x_M(t, x_0) = c^T x_M(t, h) + c^T x_M(t, z_o) = d + c^T M^{J_M} h t^{j_M}/(j_M!) + \text{H.O.T.s} + x_M(t, z_o)$. For $j < \bar{j}$ and $z_o \in \mathcal{N}^{j-1}_{c,M} \cap (\mathcal{N}^j_{c,M})^c$ the sign of $c^T x_M(t, x_0) - d$ for all small t is determined by the signs of $c^T M^j z_o$, $M = A, B$. This is the same situation as Σ_O and the result follows. For $j = \bar{j}$ select $\|z_o\|$ so large that $c^T M^j z_o$ determines the sign of $c^T x_M(t, x_0) - d$, $M = A, B$, for all small t. $\diamondsuit$

Lemma 4.3 *If Ω_O is well-posed, $j_A = j_B = \bar{j}$ and $\text{sgn}(c^T A^{\bar{j}} c) = \text{sgn}(c^T B^{\bar{j}} c)$.*

Proof: (Outline) Assume $j_A < j_B$. Since $\mathcal{N}_{c,B} = 0$ we can select $z_o \in \mathcal{N}^{j_A - 1}_{c,B} \cap (\mathcal{N}^{j_A}_{c,B})^c$ with $z_o \neq 0$. Then $z_o \in \mathcal{N}^{j_A - 1}_{c,A} \cap (\mathcal{N}^{j_A}_{c,A})^c$. Select $\|z_o\|$ small so that for all small t the sign of $x_A(t, x_o) - d$ is determined by the sign of $c^T A^{j_A} c$ and the sign of $c^T x_B(t, x_o) - d$ by the sign of $c^T B^{j_A} z_o$. By replacing z_o by $-z_o$ we contradict well-posedness. $\diamondsuit$

4.1 Necessary and Sufficient Conditions

Now we put the results of the previous lemmas together to obtain a set of necessary and sufficient geometric conditions for the well-posedness of Ω_O.

Theorem 4.4 *Assume (c^T, A) and (c^T, B) are observable and that either $Ac \notin \mathcal{N}_{c,A}$ or $Bc \notin \mathcal{N}_{c,B}$. Then the bimodal system Ω_O is well-posed if and only if each of the following conditions is satisfied:*

1) $j_A = j_B \stackrel{\Delta}{=} \bar{j}$;

2) $\text{sgn}(c^T A^{\bar{j}} c) = \text{sgn}(c^T B^{\bar{j}} c)$;

3) For each $0 \leq j \leq \bar{j}$, $\mathcal{N}^j_{c,A} = \mathcal{N}^j_{c,B} \stackrel{\Delta}{=} \mathcal{N}^j$;

4) For each $z_o \in \mathcal{N}^{j-1}$ *with* $z_o \notin \mathcal{N}^j$, $\text{sgn}(c^T A^j z_o) = \text{sgn}(c^T B^j z_o)$, $0 \leq j \leq \bar{j}$.

Proof: Necessity has been demonstrated in the previous lemmas. The proof of sufficiency is similar to the proof of Theorem 3.5. $\diamondsuit$

Generically, (c^T, A) and (c^T, B) are observable, $Ac \neq 0$, $Bc \neq 0$, and $j_A = j_B = 1$. In this case Ω_O is well posed if and only if $\text{sgn}(c^T Ac) = \text{sgn}(c^T Bc)$, $\mathcal{N}^1_{c,A} = \mathcal{N}^1_{c,B} \stackrel{\Delta}{=} \mathcal{N}^1$ and for each $z_o \in \mathcal{N}^0$ with $z_o \notin \mathcal{N}^1$, $\text{sgn}(c^T A z_0) = \text{sgn}(c^T B z_o)$. The first and third conditions are robust with respect to small arbitrary changes in the matrices A, B and c, but the second is not. However, the second condition arises by taking $\|z_o\|$ large in Lemma 4.2. If $\|z_o\|$ is restricted to be smaller than a bound B determined by the size of $c^T Ah$, then the second and third conditions are not necessary. Hence, generically, well-posedness of the system Ω_o is not robust with respect to small *arbitrary* perturbations of A, B and c. However, it is robust for initial conditions in a neighborhood of h.

References

[1] Jun-ichi Imura and Arjan van der Schaft. "Characterization of Well-Posedness of Piecewise Linear Systems" *IEEE Trans. on Auto. Control,* Vol. 45, No. 9, Sept. 2000.